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REPORT

ON THE

STUDY OF MATHEMATICS ON THE CONTINENT OF EUROPE

BY

PROF. W. H. YOUNG

SC.D. (CANTAB), F.R.S.

UNIVERSITY OF CALCUTTA
2014

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Introductory Remarks

The celebration of 150th birth anniversary of Sir Asutosh Mookerjee has, acquired, by now, an impression of not being a run-of-the-mill affair. The seminal daily diary of Asutosh Mukhpadhyay plus an anthology of his contributions in mathematics *per se* have already come to stay as bedrocks for perception and appraisal of the image of Sir Asutosh Mookerjee. A striking and an equally significant add-on to these works is the Commemorative Volume brought out on this occasion. The Convocation Addresses delivered by Sir Asutosh Mookerjee as the Vice-Chancellor form the vital part of the volume in that it seeks to unfold the thought-processes of Sir Asutosh, who kept on acquiring the ability to come to grips with the dynamics of an evolving educational citadel. Even a quick glance at these addresses ought to reveal Sir Asutosh's innovative approach for turning a stale examining body to an educationally vibrant system. There was hardly any place of thought where his mathematical mindset didn't find outpourings. As a matter of fact, he was passionately seized of mathematical pursuits in classroom contexts and research realms, too. As is well-known, he was looking for an upsurge of a nexus in mathematical teaching and learning through the emerging university system. As a mathematician, he developed unique traits capable of foreseeing an interwoven framework with the quintessence of the then mathematics in the western world. In particular, the French and British contributions had greater prevailing influences than their counterparts elsewhere.

In his Convocation addresses, it was more than a mere ritual to mention about leading figures of the western world to participatingin the restructuring the University of Calcutta. It was almost a ceaseless endeavour with him to look for personalities who would share his aspirations. He could thus find in Professor W H Young, an entrepreneur for revitalizing mathematical studies in the University. Professor Young, being the Hardinge Professor of Higher Mathematics of the University of Calcutta in the twenties of the last century, was entrusted by Sir Asutosh Mookerjee with the task of presenting a study of mathematics on the continent of Europe. Doesn't it show convincingly, the receptivity of Sir Asutosh Mookerjee to frontier areas ? Asutosh envisioned a new genre of educational agenda for the University of Calcutta. Unfortunately, neither Professor Young nor Sir Asutosh could have opportunities of a dialogue on the report prepared by Professor Young. Somehow or the other, there is nothing in history to tell us whether followup activities could be undertaken under the aegis of the university. The report could be resurrected, after a long time, from the family of Professor Young. Way back in' 1976, the University had a publication on this score. Perhaps, an occasion has arrived where this attempt by Professor Young can be taken up as an area of pursuit on changes in mathematical themes. In a way, Professor Young was a precursor of what is being talked about for the last few decades as the discipline of mathematics education. The fundamental issue possibly stems from reality whether rarely studied elements in mathematics education belong to educational systems and how they respond to change. If the focus is one change, there has to be a corresponding change in the curriculum using technology for equipping people from diverse layers of attainment and motivation. Asutosh hardly refrained from providing opportunities to excluded groups of students. Can we then afford to bring to the fore another dimension of an personality of Sir Asutosh as a leading mathematics educator, on the occasion of 150th birth anniversary? One should rekindle his image by what he could have outshone others in, exploring some ilks of mathematical study and research. Hopefully, there lies the rationale of publication of this book, unleashing new vistas of educational research in the field of mathematics.

I must express my thanks to professor Suranjan Das, Vice Chancellor of Calcutta University for kindly appreciating the proposal to republish this book. The University Press community deserves to be praised for bringing out the book within a short period.

31.12.2013
Kolkata

D K Sinha
Formerly, Sir Rash Behary Ghose Professor of
Applied Mathematics, University of Calcutta

CONTENTS

Page

Foreword by Prof. P. K. Bose, *Pro- Vice-Chancellor for Academic Affairs, Calcutta University* iii

Introduction by Prof. Guinness and Prof. Tanner vii

The Report on the Study of Mathematics On the Continent of Europe 1

Inaugural Lecture–Delivered before the University of Calcutta, December, 1915. 255

PROF. W. H. YOUNG, SC. D. (CANTAB), F.R.S.

FOREWORD

During my recent visit to the United Kingdom, I met Prof. Grattan-Guinness of Middlesex Polytechnic. While discussing with him, he disclosed that a galley proof of the Report On the Study of Mathematics on the Continent of Europe in the early part of the 20th century written by W. H. Young, Sc.D., F.R.S., formerly Hardinge Professor of Pure Mathematics, Calcutta University, might be available from his daughter Prof. R. C. H. Tanner Young. I thought as a document this report would be of great interest to the academicians throughout the world. I met Mrs. Tanner and requested her to hand over the report prepared by her father Prof. Young on behalf of the Calcutta. University. She expressed her great satisfaction at this proposal and immediately handed over the copy to me. Later on Prof. Guinness and Prof. Tanner were kind enough to write an introduction to this book.

The Syndicate of the Calcutta University had asked Prof. Gaston Darbouse, the doyen of French mathematicians, to suggest a suitable person to found the new department of Pure Mathematics. At this suggestion Prof. W. H. Young was appointed the first Hardinge Professor of Pure Mathematics in August 1913, at a salary of £1,000 per year.

After sometime the Senate and the Syndicate of the Calcutta University decided to accord the necessary facilities to Prof. Young to tour Europe for the purpose of completing and renovating his knowledge of academic matters on the content.

Later on this tour was extended to other countries such as America, China and Japan. On his return he submitted this report which the University wanted to publish immediately. By the time the galley proof was ready. Prof. Young left Calcutta University to accept an assignment in the University of Wales. The proof was sent to him but this matter was not further pursued.

In 1913 an unfortunate controversy arose between the Syndicate headed by the Vice-Chancellor Asutosh Mookerjee and Mr. Bampfylde Fuller, the Lieutenant-Governor of Eastern Bengal and Assam in regard to the affairs of a school. The Lieutenant-Governor even threatened to resign the Chancellorship of the University. Controversy with the Government of India arose in regard to three appointments to the postgraduate department of Arts. Thus did the woes of the University begin. Inspite of great financial handicaps, Sri Asutosh was firm to develop the Post-Graduate teaching and research in the University. On 27th September, 1913, he gave an account, before the Senate, of the teaching arrangement in the Post-Graduate departments of Arts and Science. In his statement he specially mentioned Professor Young, a fellow of the Royal Society of England as the Hardinge Professor of Pure Mathematics. He was not only keen to appoint him but also he was responsible for sending him to various countries in order to develop the newly founded department of Pure Mathematics inspite of acute financial stringencies. This clearly shows the vision of Sir Asutosh for establishing the Post-Graduate teaching and research in the University.

Before I conclude I record my most sincere thanks to Prof. Guinness and Prof. Tanner for their interest in this matter and to Shri Arun Roy, Registrar, Calcutta University without whose active help it would have been impossible to publish this book.

CALCUTTA UNIVERSITY, P. K. Bose.
2.11.1974

INTRODUCTION

This book contains the text of a report on the teaching of university mathematics in Europe and elsewhere, prepared by William Henry Young (1862-1943). The circumstances of its preparation have been described in detail elsewhere.[1] Therefore we need recall here only the principal points.

Young and his wife Grace (1868-1944) formed one of the most productive partnerships in the history of mathematics.[2] During nearly thirty years of collaboration they produced between them three books and over two hundred research papers, chiefly in the foundations and applications of mathematical analysis.[3] However, this book is related not to their creative work but to Young's career as a professional mathematician. In this career he was less successful than in his research, for he obtained only a few appointments, each one for short periods. One of these jobs was as the first Hardinge Professor of Mathematics at the University of Calcutta. He took up his appointment in the autumn of 1913, and soon realised that the creation of a department of mathematics at Calcutta, and the development of university mathematics 'elsewhere in India, was hampered by the authorities' lack of experience and acquaintance with the problems involved. So he proposed to Lord Hardinge (the Viceroy of India) that the University finance visits by him to Universities in Europe and elsewhere, so that he could obtain at first-hand information on the conditions prevailing there.

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[1] See. I. Grattan-Guinness, 'University Mathematics at the turn of the century. Unpublished recollections of W. A. Young', *Annals of Science,* 28 (1972), 369-384. This article contains an annotated edition of parts of the report.

[2] For a biography of the Youngs, see 1. Grattan-Guinness, 'A mathematical union : William Henry and Graco Chisholm Young', *Annals of Science,* 29 (1974), 105-186.

[3] See. I. Grattan-Guinness, 'Mathematical bibliography for W. H. and G. C. Young,' *Historia Mathematica* (Forthcoming).

The proposal was accepted, but the start of the First World War in the Autumn of 1914 prevented the visits and hindered the preparation of the report. Eventually he made one trip to Japan and America in 1915, and another during 1916 which included some European countries. He obtained other information from correspondence with prominent mathematicians of his acquaintance and from recent publications of the International Commission on Mathematical Education. In the case of England, he drew on his personal experience.

He seems to have written up the report during 1916; but for health and other reasons he terminated his appointment at Calcutta in 1917. The report did not advance beyond the set of printed and partially corrected galleys which he kept among his papers.

One of us, R. C. H. Tanner, the Youngs' eldest daughter, has been responsible for the preservation and organisation of their collective papers since their death, including the annotated set of galleys of the report. It is from these galleys that this book has been edited for publication under the direction of Professor Bose.

SEPTEMBER, 1974

MRS. R. C. H. TANNER AND
I. GRATTAN-GUINNESS.

REPORT
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Various circumstances have prevented this report being complete. The war had already commenced when I received word from Calcutta that the Syndicate and Senate had been good enough to accord me the leave necessary to enable me to return to India. An extended tour–such as I had proposed to myself–for the purpose of completing and renovating my knowledge of academic matters on the continent of Europe was no longer possible. It is moreover even in neutral countries such as Switzerland not easy to get help in acquiring information, owing to the fact that most able-bodied men are mobilised and others have special additional duties conferred on them.

It has been accordingly impossible to deal properly with the "allied subjects." I have not dealt adequately with the relations of Mathematics to the various other branches of knowledge over whose development it is beginning to exert an influence.

Further the scope of the enquiry proposed to me did not include the institutions of Japan and America, and in particular the new Post-graduate

Colleges of the United States. One difficulty can only be touched upon.

In a report of the kind contemplated something more than a mere enumeration of facts is obviously desirable. The advantages of one system and the disadvantages of another need to be brought out, if the report is to have its full usefulness. But it is clear that there is some danger of praise and blame being taken amiss at a moment like the present. Two systems, it might be urged, are fighting for the mastery, you cannot praise one even for an achievement of the second order of importance, without furnishing it with an arm which its adherents would hasten to use, and the excellence of a University or of a system of educatism is no unimportant factor in judging of a civilisation.

In point of fact, however, my experience of neutral opinion has led me to conclude that, certainly in Switzerland and America, and I believe elsewhere, the feeling has been steadily growing that the present moment it is a still bigger question than the future of Universities which is at stake, namely that of the right of the individual as against the tyranny of a military caste. On the other hand, indications are not wanting that even in matters academic the future may well be with the Americans, the Russians and as well as with the British and the French. The great lesson which Germany has had to teach, the necessity for an improvement in the organisation of research and in its equipment, was learnt indeed some years since in America, and is fast being learnt elsewhere, and the second great lesson that examinations should not be allowed to trammel the development of intellectual life is in process of being learned all over Western Europe. Some contend indeed that German Universities have for the moment passed their climacteric in my own subject we do not now look specially to Germany, and it is to mathematicians of other countries, some but not all of whom have owed much to Germany, that the younger mathematicians have begun to look for inspiration.

For various reasons then it has seemed to me, at any rate in this preliminary report, best to content myself with a somewhat impersonal account of mathematical studies. I have not always given my own opinions but have confined myself largely to descriptions of the systems prevailing in the principal countries of Europe. It has been convenient however to allude to certain tentative proposals which I conceived might be made but I am obliged to defer the consideration of details on some of the points at issue. With regard to equipment I strongly advise reference to America. It is possible that I may myself be able to give the further information which is required, if I am able to re-visit that country shortly. I am also strongly of opinion that the Syndicate should obtain an account of the Japanese Universities and I should be myself prepared to report on them if desired to do so, at any early date in the event of my being authorised to return to England, *via* the far East.

The success of a school of Mathematics is necessarily dependent on a variety of conditions being fulfilled. Among these its relation to Government, and the nature of the freedom which it possesses; the mode in which its professors are selected, and the position which it is able to offer to them, are no less important than the character of the students, the organisation of their studies, the facilities afforded to them, and the activities which are opened to them after leaving the school. Properly speaking then an enquiry into the characteristics of the mathematical schools of Europe and America cannot be said to be complete unless all these matters are dealt with.

The information contained in the present report, which deals chiefly with programmes and methods of instruction and examination and the titular rewards obtainable, cannot be expected to furnish the key of itself to the success of certain schools of mathematics and the failure of others.

Further even if the terms of the resolution of the Syndicate were such as to have suggested to me the larger sphere of enquiry, and the time and opportunities for carrying it out had been available, the Report would have then at best corresponded to a dissection of the inanimate corpus of the mathematical world. Something more than this is needed if we are to understand its physiology as well as its anatomy and more still if we are to represent to ourselves precisely the kind of existence it enjoys.

It is almost impossible without visiting Universities oneself and living for a considerable time in the milien which they afford, to understand the secret of their vitality and their success in respect to anyone or other of a group of studies, and it is no less difficult to explain to others what one has oneself seen and experienced.

If a great University is to be built up, each individual school of thought must have as founders, or as renovaters, men who have themselves been outside the Cave of Local Prejudice which necessarily exists in all parts of the globe. This is the lesson which the study of the great universities of Europe and America teaches us. It is right that such men should be asked to give an account of what they have seen, but unless a large measure of confidence is given to them, the desired result is not likely to be obtained. It is almost certain that in some vital points, their opinions may be overruled. They may indeed, even voluntarily defer to the opinion of others, persuaded by what they hear as to the difficulties in the particular environment. After all, with the best men, impressions are often of greater value than fully reasoned out conclusion. We have felt that such and such a system was superior, and yet we may be temporarily argued out of the faith that is in one that it should be followed as a model.

One important remark must be made illustrating the necessity of actually visiting the universities, and living for a sufficient period in the milieu they afford, in order to judge the advantages and disadvantages of the system of which they are an outcome. Even detailed accounts of the subjects of the courses and enumeration of the institutions which the student has the opportunity of joining, may be entirely misleading. Sometimes indeed one is tempted to ask whether a man who has attempted a report of such and such university, has really had the courage to portray things as they are. In any case I have certainly acquired the conviction that success has not always crowned his efforts.

A mere skeleton list of lectures tells you nothing. This list may include all the lectures which have ever been given for a period of twenty years. On the other hand if it definitely relates to a particular year, some of the lectures may have been announced and not given, and, when given, may have been of an extremely unsatisfactory nature, not merely from the pedagogic, but also from the scientific point of view. At best such a list may merely show what the Faculty think should be represented, or what the individually isolated teachers are interested in.

One of the characteristics of English life, one responsible partly for its charm, but none the less to some extent a sourse of weakness, is the credit not infrequently allotted to amateurs. Sometimes this credit is well-deserved, but it would be the greatest possible mistake to suppose that it necessarily follows that, because a set of lectures is announced at an English University, they at all correspond to what would be tolerated on the Continent of Europe. Elsewhere than in England a man is almost sure of severe critics, at any rate both in France and Germany the standard is very much higher.

But it is true, not only in England, but also elsewhere, that the subject of a set of lectures, and even the enumeration of the topics treated of, gives very insufficient information as to the actual ten our of the treatment, and the enormous differences such as those which exist between the treatment of the Calculas by Picard, Goursat, or de la Vallee Poussin, Lebesgue and that of an ordinary teacher of Mathematics at an English provincial university might pass almost unsuspected on a cursory inspection of programmes.

Again the mere study of printed matter relating to the University of Paris would scarcely reveal the existence of its great defect. The professors and students are alike isolated among themselves, and between them there is a great gulf fixed. What a student learns from attending a course of University Professor at Paris he learns by his own unaided effort, and in spite of difficulties. He does not know, and is not known to the professor personally. And it is, I believe, quite unusual for the foreign student in Paris to get any definite lasting benefit from his attendance at such a course. On the other hand in no University programme published in Germany is the fact mentioned that students come almost invariably into close contact with the professor, are known to them personally and invited to their table, and that one or more of the professors in each subject, or branch of a subject, is in the habit of setting apart a definite period or periods every week, during which he may be consulted by the students about their work. That moreover the Doctor (Dissertations of the students are almost always the outcome of work undertaken immediately under the eye of the professor, to whom the student has had, during the period of preparation an amount of access limited only by the number of students and the number of free hours at the disposal of the professor.

These and other remarks which might be made would sufficiently show the grave danger which exists in attempting to draw up plans for a University based on information acquired from university programmes or official Blue Books.

In this same connection, the spirit in which the professor does his work depending as it does, on the position which is afforded to him and the freedom which he possesses must be taken into account.

In Wales a professor has been little better than a servant. He has had the wages of a clerk, and has had his movements outside of the University restricted. A local tradesman, a member of the council of a Welsh College, has been known to say that he was prepared to insist on an exact account being presented to him of every hour in the professor's day. What time he spent in lectures, what time in preparation, and what time in looking over the exercises of his pupils, examination papers and so forth, and what time he had unoccupied. He was determined to see, he said, that the professor earned his money. Even in England the pay of a professor is at times less than one third that of the Vice-Chancellor, at those provincial universities where the latter is a paid official.

In Germany, on the other hand, a professor from one day to the next may be summoned to Berlin, and invited to the presence of the Emperor. In his own University town, his position is the highest conceivable, and even the bourgeoisie, the small as well as the bigger tradespeople, take a pride in their eminence, and an interest in their activities.

In Paris, again, a professor is a personnage, one who must be approached with all possible consideration and possessed of a very considerable amount of power and influence.

Elsewhere than in France and Germany the conditions are varied. Notably in Italy grave fears are being expressed as to the future of university and other advanced instruction, owing to the relatively low social position and pay accorded to a professor. The engineers are regarded as possessing a higher social status besides of course enjoying, as the more successful engineers do all over the world, a larger income. Even in Italy however, the Minister of Education is commonly a former professor, and among the Senators of the kingdom there are always a certain number of distinguished professors. This contrasts very favourably with the system prevailing in Australia, where professors have to bind themselves by an oath to hold themselves entirely aloof from politics, or any intermingling with the affairs of Government.

My contention is then that an argument that a particular system of university organisation in a country B is better than in a country A because better original work is produced in B than in A may be a dangerous one. All the various factors have to be taken into consideration. In no case can bricks be made without straw nor without competent brickmakers.

FIRST ROUGH DRAFT OF PROPOSALS WITH REGARD TO THE EXAMINATIONS

Preliminary Remarks

India possesses at the present moment, a number of institutions of different grades. This which might seem a disadvantage, may be, I conceive, turned to an advantage. In most parts of Europe, or at least in several of them, the period of two or three years previous to entering the University is often spent unsatisfactorily at school. The organisation of the school is necessarily taken up with the classes below the two or three highest classes. And the work in the highest classes not infrequently suffers in consequence. The pupils are too old for the ordinary school life and too young for the modern university at its best. In England pupils in these classes are often receiving special individual instruction which makes great demands on the staff. It is indeed of the nature of coaching. Moreover even in the best schools the members of the staff are not always competent to do this specialised work. In Switzerland and to a less extent in Germany, some of the more intelligent boys undoubtedly waste a certain amount of these last few years through having to wait for their duller comrades. In France the rules are not so rigid and there is a special organisation of classes in this higher work.

India might do still better than France by setting aside a special class of institution for the two years after the present Matriculation Examination. The work done in these two years corresponds to the work done in the schools of Europe. The matriculation should *not be regarded as* a university examination, but as an entrance examination into these colleges. The residential character of these colleges might or might not be wholly or partially insisted on, and the maintenance of the students at these colleges might or might not be defrayed wholly or in part by the State, or this might be done in the case of a specially selected number of peculiarly gifted boys. *The examination should in any case be partly oral,* and should be conducted under the general guidance of of the university at the school and not at the college, if possible. It should be the special duty of the university to see that the knowledge of English possessed by the pupils should be sufficient, and it should be one of the special duties of the colleges to see that a thoroughly satisfactory knowledge of spoken English, both as regards speaking and hearing, was gained during these two years.

In this way the present university course of six years up to the degree of Master of Arts, would be relieved of the dead weight of the first two years.

In point of fact, at no university of Europe is an university course of six years contemplated even distantly, throughout which the main object of the student is that of passing some examination. In Germany a student may very well be six years at the university before taking his doctor's degree, but this will be exception, even in the case of a student who could remain to remain there so long, and the mental training undergone at such a University is entirely different from that intained by preparation for a series of examinations. It will be seen, however, that the first two years at Calcutta University as at present constituted correspond, not to two years spent at an European university, but to two years spent at a Gymnasium or Lycêe or other corresponding school.

While the present Matriculation Examination might well be called College Matriculation, the present Inter. Arts and Science Examinations might be fused into one called the University Matriculation Examination.

It would appear that some process of the kind here advocated has actually gone on in France. For the examination which in Swizterland is called the Maturité, German the Reifeprüfung or Abiturientenexamen, and in England the School Leaving Examination, is in France called the Baccalaureat suggesting that this was once a University examination conferring a degree. The corresponding degree has entirely disappeared from the French University programme.

As regards the details of the examination now to be called the University Matriculation Examination, this perhaps might be rendered more homogeneous by abolishing the distinction between Pass and Honours, and possibly by replacing the Honours Examination by additional Special Papers, success in which might confer certain privileges, or some emoluments. But with regard to this question of detail I am not prepared at the moment to express a definite opinion. It is in any case essential that the examination should be in part oral, and held, if possible, in part in the college at which the student has been studying, either as an inmate, or otherwise.

The great advantage of the student being an inmate of such a college, apart from the more obvious ones, is the fact that books would be accessible to him without his having to buy them. It is one of the great disadvantages of a boy remaining so long at school in Europe that he commonly does not learn to use a library till he is nineteen or more years of age, and has become a member of a great university. In certain cases it is already too late for him to learn, and the art of using books is never acquired by him.

The University should in no case have to provide text-books or other books merely required at this early stage of students career, and not later.

Taking the examinations in order, we next come to the present B.A. or B.Sc. Examination. A few privileged colleges specially well-equipped and staffed might expect to retain their students to a certain extent at least during the two years preparation for these examinations. They would correspond to the Ecole Normale Superieure in Paris, but their students would attend a certain number of lectures at the University, while at the college the lectures would take the form more of the French Conference, with questions and exercises. It is desirable however that the distinction between Pass and Honours should be abolished, or that they should not be rewarded with the same degree. What is understood by B.A. at Oxford or Cambridge, is much more that the holder has spent the recognised time and has spent a reasonable amount of energy in reading a certain number of books or attending a certain number of lectures. It certainly does not convey the idea that the holder is in any sense of the word a well-informed person. It is merely an accompaniment of the passing of the examination, whatever it may be. In all the questions of appointment of an Oxford or Cambridge the question is "what examination has to be passed, and how?" The fact of his being a B.A. of Cambridge is merely equivalent to the statement that he is a Cambridge man, and has consequently passed though certain social processes, due to the residential system. In no country, I believe, outside England, there is anything corresponding to this distinction of Pass and Honours, and though the Oxford and Cambridge plan has been followed in the other Engligh Universities, the best professors, notably in Liverpool, are against it, and do their utmost to minimise its effect, pending its final removal. It is noteworthy that the womens' colleges even at Oxford and Cambridge themselves, either refuse to take or strongly discourage the existence of Pass students. Some of the men's colleges indeed take the same line. It is not too much to say that, but for the question of fees (and sports in certain cases), and the fear that the incomes of the colleges would be seriously affected, Pass degrees would be entirely abolished, both at Oxford and Cambridge.

It appears to me that the difficulty at Calcutta could perhaps be overcome by adopting the French system of the Licentiate. The weaker, or less well-prepared students would be sufficiently allowed for by the regulation following him to take his various certificates at different times.

As regards the details of the preparation for the examination, and the examination itself, attention should be called to the fact that, even in France, whose system, as we have seen approaches most nearly to the English system, the students for the Licentiate are encouraged from the first to read original papers; *they are examined orally as well as "par écrit,"* and the written examination is of quite a different type to that in vogue in Calcutta and in England. A small number of problems are set, which have to be solved completely and a considerable time (seven hours for each paper) is allowed to each candidate. The demand on the memory is very much less, and there is no possibility of accumulating marks by answering a large number of easy questions more or less imperfectly.

Passing to the M.A. and M.Sc. Examinations there is nothing corresponding to these outside the British Empire. No mathematician of my acquaintance would allow the desirability of the third and fourth years of proper university work being spent exclusively in preparation for a written examination.

If my points of view were accepted, these two years would be spent exclusively at the University, and not in one of the colleges at all. They are too important to be risked, even in part. They should be employed by the student in gaining an acquaintance with the thoughts of great writers in the original. They desire to produce something new himself, to extend the bounds of his science should already have become part of the student. The process of attempting merely fo assimilate and reproduce the work of others, if continued through these two years, is bound to have a deadening effect on the mind. The powers of imagination and initiative (which every mind possesses to a greater or lese extent if are made use of in time), will fail a student whose mind is cribb'd, cabin'd and confined in the examination system for so long a period. To "understand" the thoughts of others is not to think, and without thinking independently oneself, a proper estimate of the work of others is impossible. At this stage indeed of his University career more than one student will 'find' himself for the first time. His very independence of mind, which may have interfered with his rapid progress hitherto, will now be his most valuable asset, and theories which before seemed obscure to him will become suddenly illuminated by the flashes of his own intelligence, while it strives, like those of this predecessors, to seize and even to create the true and the beautiful.

Moreover if the step has not been taken by the student at this stage, material difficulties will usually prevent him taking it later. Limitation of time and space in India as elsewhere, and perhaps even more than elsewhere, will prevent his embarking on the ocean of independent thoughts, and he is likely to sink into the mere slavery of routine, meticulous in small things and indifferent in great. What I am quite clear about is that the student should during these years be under the charge of professors who are not concerned with any other work, but able to devote themselves exclusively to their own researches, and those of their pupils.

I write of course in only partial knowledge of cause. It may be that at the present moment, when the difficulties at the earlier stages of an Indian's education are taken into account, that the time allowed up to this stage is not sufficient, and that three and not two years are needed after the B.A. for the present at least. That is, however, from my present point of view, merely a question of detail, though, if generally recognised, this difficulty might have considerable normal consequences, involving even the entire change of the relation of the present M.A. and B.A. Examinations to one another. Thus, though I do not advocate this, it might be deemed advisable to suppress the present M.A. and replace it by an examination subsequent to the B.A., at an interval of one year from it, this examination bearing the same relation to the B.A. that the Agregation does to the Licentiate in France. The actual degree of M.A. might then be suppressed, and two years might be given after the Agregation to obtaining the Doctor's degree. In this way only one additional year would be added to the existing course at the end, the total University course, properly so called, in the case of a student proceeding to his Doctor's degree, would be five years. I should expect that under this system proper modifications having been introduced throughout the University course, that it would become the rule, rather than the exception, for the really bright students to take their Doctor's degree.

The standard for the Doctor's degree might with advantage be that which obtains in Germany, rather than the higher one of the French.

The alternative plan of securing for the Doctorate a higher standard, equal to that of France, could only be secured by increasing the difficulty of taking the degree of Master of Arts; giving it, in fact, for much the same kind of work as gains the Doctor's degree in Germany. The Indian Doctorate would then share with the French Doctorate the perhaps undesirable distinction of being almost unique as regards difficulty of attainment.

Additional Considerations

It does not seem desirable to seriously increase the difficulty of the present Matriculation Examination. By keeping this, which would in the present scheme become the College Matriculation Examination, at a moderate degree of difficulty we ensure a large entrance of Indians into higher institutions. In the present state of Indian education it is impopossible–and it is for reasons referred to above probably undesirable in any system–to provide in the ordinary secondary schools for higher work than this examination requires at present. On the other hand it is essential that the examination should be partly oral, even if part of the questions had to be asked *viva voce* by the masters at the school in the presence of suitable external representatives. The introduction of these oral questions in the schools at the College Matriculation Examination would have the excellent effect experienced all over Europe of encouraging all through the school course this kind of test. Further when coming to the College, and subsequently to the University, the habit of answering questions *viva voce* having been previoulsy acquired, the students would take the oral part of their subsequent examinations as a matter of course, and experience no difficulty with it.

Attention cannot be too strongly called to the fact that the practice of dispensing with oral examinations is absolutely unknown on the continent of Europe, and appears to me to be also in entire disaccordance with Indian traditions, relating to times before the arrival of the English. In some European countries the necessity for an oral examination is so strongly felt, that written tests have indeed been too little utilised. The English practice is now at the opposite pole. In my own undergraduate days the oral examination still lingered on, but it was already dying. I am afraid there is no doubt that the convenience of the examiner, far more than the good of the candidate, was the main cause of its falling into absolute disuse.

As regards the details of the College Matriculation Examination, this contains no Drawing at all. In every European system with which I am acquainted either by Drawing or otherwise the pupil's grasp of three dimensional relations is trained all through the school course. In an ideal system the Matriculation Examination would certainly contain a Drawing test, with a low Pass standard, no doubt, and the Geometry paper would include certain questions, not taken from Euclid's XI the book, but requiring for their satisfactory answering the power of picturing for oneself the similar solid figures. An eminent German scholar has gone so far as to state to me his belief that Indians are incapable of Geometrical insight, that they cannot think except in the plane, and then only with paper before them. This is of course absurd, all depends on training. To think in three dimensions is *more* natural than to think in two, and the exclusive practice thinking of drawing as figures on paper, instead of seeing surfaces and lines in space, is the outcome of an imperfect appropriation of the gifts of civilisation.

With some diffidence I venture to suggest the use of a book by my wife and myself in the schools of India, a book which has been translated into German, Italian, and used in the schools, entitled the First Book of Geometry. Reference to the only Indian book* on Geometry with which I am acquainted appears to show conclusively that this First Book of Geometry would be consonant to the Indian genius.

With regard to the limit of age which is at present sixteen on the first day of the month at which the student appears at the examination, this minimum appears to be too high. Exceptional boys in England and in France would be able to pass an examination of this degree of difficulty at 15, or even earlier. It was natural that this age limit should be imposed at the time these regulations were made, but if the proposal of this Report be adopted of regarding the present Matriculation Examination as a preparation for a College course, itself preparatory to a University one, the lowering of the level to 15 years of age is called for.

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* Sundara Row's modest but interesting little volume on Paper Folding.

If the present Matriculation Examination is regarded as closing the ordinary school course, and preparatory to College study, it would naturally be slightly modified. There is not the same reason for giving a choice of subjects. The work could moreover be lightened by having the examination entirely altered in one or more of the subjects. Under the present regulations it is possible for a boy to leave school apparently, or at any rate to present himself at the examination, without possessing any knowledge of History or Geography at all. There is nothing precisely corresponding to this examination, as it stands, anywhere on the continent of Europe.

The defect of the examination in English is that it is entirely written. It is treated in the examination as a dead language. It is however of importance, even at this stage, that the Indian student should be able to understand, and make himself understood, in discussing any of the school matters with a stranger.

I understand that part of the difficulty experienced by the Indian student in acquiring English is due to the fact that the Government regulations do not allow him to begin English at a sufficiently

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early age for the process of acquisition to be an easy one. I have found with my own children that it is essential that the *second* language should be begun conversationally as early as possible, certainly not later than $7^1/_2$, and preferably at 5 or 6 ; the ground work then laid is one on which it is easy to build. It is however important that conversation in the language should go on every day for at least half an hour, or at least on five or six days of the week. As regards the whole character of the examination, I am of opinion that the papers are too long, and the special effort required in preparing for the examination so as to be sure of passing is likely to be such as to have an effect on the schools which would be far from being wholly good. I presume that the passage from one class to another of an Indian Secondary School involves the satisfying of certain tests. If this be so, it ought to be possible to dispense with questions on Multiplication and Division, for example, in the Matriculation Examination. It is unthinkable that questions of this kind should be asked in an examination at this stage in a continental school. On the other hand the paper in Mechanics seems to be in general of much too advanced a nature. It would not be unsuitable for the proper University Matriculation, which in this Report is taken to correspond to the Intermediate Arts or Science. It appears to be entirely out of place in its present form at the stage of the present Matriculation Examination. In the schools of Europe only boys in the highest class would be able to make anything of it. A student who is able at the entrance into Calcutta University to deal *intelligently* with such a paper as was set in 1912 ought at the end of six years to have achieved original work of a high character, and be already counted as a scholar of some standing, to be in a very different position, in fact, to that occupied by the present successful M.A. or M.Sc. candidate.

I do not know what facilities Indian Schools possess as regards the experimental treatment of Elementary Mechanics, but I have been not incredibly informed that Indian students are not usually strong on the practical or mechanical side, and that they'are best in dealing with abstract thought. I naturally therefore surprised to find so difficult a paper on Elementary Mechanics set at the Matriculation Examination, and this more especially as I believe that no knowledge of Mechanics and Physics is of value that has not been built up on individual observation of natural phenomena, and the various devices of which man has availed himself for utilising natural forces.

With regard to the programme of the present Matriculation Examination in Geometry, it appears to be that which I helped to draw up for Cambridge University more than ten years ago. This is not the place to explain the reasons which led me to take an active part in drawing it up. It is important however to notice that the Cambridge Little Go was devised for residential students having a low Pass standard, far lower intellectually than any University students on the continent I have ever encountered or heard of. On the other hand this examination has been more and more replaced by the School Leaving Examination (a more difficult one) even in England. But it is clear that as soon as the Examination becomes one which is held wholly or in part at the School the character of the Examination at once changes. The object is merely to discover if the pupil is fit to go on to the next stage, and, as the examination is partly oral, the risk of error is diminished. Moreover as the pupil is freed from the necessity of rubbing up the elementary part of the subjects, and relieved from the worry thereby entailed, he is able to do somewhat higher work than would otherwise be possible.

I take the example of what goes on in the College de Geneve, the great public non-resident school of the Canton and Republic of Geneva, which holds the premier position among all the schools of French-speaking Switzerland. It may be regarded as a modification of the French Lycées etc., of such a kind as to bring it into conformity with the German Gymnasium, Real-Gymnasium and Oberrealschule. Like other Swiss schools it differs from the German in aiming rather at securing a good education for the average pupil ; the more intelligent pupils are somewhat kept back for the sake of the less intelligent, whereas in Germany the average pupil is required to make great efforts as the standard to be obtained is higher than that which is natural to him. The system at the Collègé de Genéve differs again from the French schools in insisting rigidly, in principle at least, on a minimum with regard to age, whereas in a French Lycée a clever boy can pass quickly through his school, and enter the University at the age of seventeen or eighteen, and even earlier. My own son is the only marked exception at the Collége de Genéve which is apparently known. He found the work so easy that a special dispensation was allowed him, but the Department of public Instruction was in point of fact not informed by the school authorities, though the former could have obtained the information, had it occurred to them to ask for it. My boy obtained the highest place in one of the four divisions of the school, and accordingly went on to the University, leaving the school at the age of sixteen years and one month, the ordinary age being three years older, and exceptionally two years older. I may say that this result was not due to overwork on my boy's part; on the contrary he worked less and not more than the other boys, and received no help outside the school. His work had been indeed so successfully arranged from his earliest years that, though all through his school life he had never been pressed in any way, he not only completed the full school course, but did various things besides, including acquiring the facility of speaking and writing equally well the three great European languages, beside a not despicable knowledge of Latin and ancient and modern Greek. I go into these details because they bear on the question as to whether the programme of the Collége de Genévé is one which an averagely intelligent pupil may be expected to master in his school career.

The College has four divisions : the division in which least Mathematics is done in the Classical

Section. I give a detailed statement of what has been accomplished by the boys two years before leaving the College, that is at the period which corresponds in my scheme to that at which they should in India take the College Matriculation Examination. I give only the last items of the course, those actually studied in the year previous to this period.

ALGEBRA. Equations of the first degree in several unknowns. Squares, square roots, roots of the second degree, quadratic equations with one unknown. Properties of the roots of such equatisms. Biquadratic equations. Equations of the second degree in several unknowns. Problems.

GEOMETRY. In the previous year the plane and the lines which can be drawn on it, the quadrilateral, the circle, proportional lines and similar figures, and the areas of plane figures had been already studied. In the year just drawn to a close the pupils have studied the plane as existing in space, perpendicularity and parallelism, dihedral angles, trihedral angles, polyhedra, round bodies.

Two years later, at the end of their school course, the period which in my scheme corresponds to the University Matriculation, or in the existing Calcutta scheme the Intermediate Arts or Science Examination, the pupils have done in the first year, and passed at the end an examination on it in the following.

ALGEBRA. Graphical representation of simple functions of a single variable. The exponential function. Logarithms considered as exponents. Progressions. Compound interest, annuities, etc. Permutations and Combinations. Elements of the

Calculus of Probabilities. The Binomial Theorem for a positive index.

TRIGONOMETRY. Trigonometrical functions. The calculation of the right-angled triangle. Reduction of angles to the first quadrant. Theorems on projection. Principal relations between the sides and angles of a triangle. Resolution of triangles. generally. Problems.

And in the second and last of the two years they have studied the following, and at the end of the year have been examined in it :–

ANALYTICAL GEOMETRY OF TWO DIMENSIONS. The point, the straight line, the circle, the ellipse, the hyperbola, the parabola. Curves of the second degree considered as the sections of a come; their ordinary equations. Exercises.

It should be particularly noticed that the examination at the end of each of the two years bears only on the programme of that year ; further that Mechanics is not included in the programme of the Classical Section, though Experimental Physics is studied in both the last two years.

If on the other hand we turn to the programme of the Section Technique, where the most Mathematics is done, we find a much more detailed programme. The Differential and Integral Calculus, Theory of Equations up to and including Sturm's Theorem, and the simple elements of Analytical Geometry of Three Dimensions have all been studied before the pupil leaves the school. Time has been spent not only on the theory, moreover but difficult problems in Descriptive Geometry have been solved.

Instruction in Descriptive Geometry extends over the whole of the last two years, and has enabled the pupil to deal satisfactorily, not only with problems on the intersection of polyhera, and the shadows cast by these bodies, on the development of the cone and cylinder, conical surfaces, etc., but much more difficult matters, such as general ruled surfaces and surfaces of revolution, including practical and theoretical acquaintance with tangent planes having more than one separate point of contact, as for example, the bi-tangent planes of an anchor-ring. This has, of course, been accomplished just before entrance to the University or Polytechnicum. Two years previously the knowledge obtained is the same as that gained by the classical students at the corresponding period with some important additions, including Indeterminate Equations, Fractional and Negative Exponents, Cube Roots, etc. Continued Fractions. Calculation of Regular Polygons. Similar and Symmetric Polyhedra.

Add to this that in all the sections, one hour a week has been devoted during the last year before entrance into the University.

It will be convenient to give the detailed programme of each of the last two years :—

Mathematical Programme of the last year but one at the College de Geneve.

(Pupils of 17 years of age and exceptionally 16).

GENERAL MATHEMATICS (Four hours) :

ALGEBRA. Integral and fractional powers. Graphical representation of simple functions, The exponential function. Logarithms considered as exponents. Applications.

PROGRESSIONS. Compound interest, annuities. Permutations and combinations. Elements of the calculus of probabilities. The Binomial Theorem with positive integral index.

TRIGONOMETRY. Trigonometrical functions. Calculation of the right-angled triangle. Reduction of angles to the first quadrant. Theorems about projections. Logarithmic transformations. Trigonometric equations. The principal relations between the sides and angles of a triangle. Resolution of triangles. The elements of spherical trigonometry.

SPECIAL MATHEMATICS. (Three hours) :

ALGEBRA. Expressions of the second degree in 3 variables. Maxima and minima; geometrical applications. Elementary theory of determinants.

GEOMETRY. Theory of transversals. Pole and polar. Radical axis.

TRIGONOMETRY. Complex quantities. De Moivre's Theorem. Roots found by means of trigonometry.

DESCRIPTIVE GEOMETRY. (Two hours) ;

Orthogonal projection. Elements of the theory of shadows. Methods used in descriptive geometry. Their application to the isometric projection of solidus, as well as to problems concerning distances and angles. Plane section, development, shadows (propre et portée) of a prism and pyramid.

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For the purpose of comparison, and also because Mechanics enters to a certain extent into the Mathematical programme, we give the syllabus of Physics also :

PHYSICS. (Three hours). *Mechanics.* Movement; force; mass; work and conservation of energy. *Gravity.* General properties; measurement; pressure; density. *Heat.* Dilatation, specific heat; mechanical equivalent of heat; fusion and solidification; latent heat; vapour; maximum of tension; ebullition; density of vapour; freezing machines; steam engines.

MATHEMATICAL PROGRAMME OF THE LAST YEAR AT THE COLLEGE DE GENEVE

(Pupils of 18, exceptionally 17, years of age.)

GENERAL MATHEMATICS. (Four hours) :

ANALYTICAL GEOMETRY. The point. Harmonic division. Transformation of co-ordinates. Representation of lines. The straight line. Curves of the second degree. Tangents to curves of the second degree. Pole and polar. Special study of the circle, ellipse, hyperbola and parabola. Equations of curves of the second degree in oblique co-ordinates and polar co-ordinates. Elements of geometry of three dimensions.

SPECIAL MATHEMATICS. (Three hours) :

ALGEBRA. Continuation of the Binomial Theorem. Series. Limit of $(1+1/m)^m$. Differential coefficient. Taylor's Theorem; development of the principal functions in power series; calculation of loggarithms. Maxima and minima; true values; indeterminate expressions. Primitive functions.

GENERAL THEORY OF EQUATIONS. Integral functions. Theorems on the number of roots of an equation of the m-th degree. Decomposition of an equation with multiple roots to several equations with simple roots. Sturm's Theorem. Commensurable & incommensurable roots. (Three hours).

DESCRIPTIVE GEOMETRY. Trihedral angles. Intersections and shadows of polyhedra. The cone and cylinder of revolution. The sphere. Plane sections. Development of the cone and cylinder. Intersections. Tangent planes. Shadows. Surfaces of cones and cylinders. Surfaces of revolution. Ruled surfaces.

PHYSICS. (Four hours).—Electricity and magnetism. Generation of electricity by friction; attraction and repulsion; Influence; potential; electric atmosphere; caloric and chemical action of a current; magnets; terrestrial magnetism; the experiment of Oersted; action of cruents on magnets and vice versa; induction; transformers; dynamos. –Acoustics. Production of sound; velocity; intensity; pitch; musical intervals; musical tubes. Optics. Propagation of light in homogeneous media; shadows; penumbra; intensity; velocity; reflexion; refraction; decomposition and composition of light.

Three hours are devoted to the above programme, the remaining hour is given to notions of mechanics, additional to those gained in the preceding year under the head of Physics.

In this class they also have chemistry, both theoretical and practical. The programme is as follows :

CHEMISTRY. (Three hours)—Chemical phenomena; elements; combination and decomposition; the notion of an atom; nomenclature; valency; hydrogen; the metalloids, metals and the salts which are most important from an industrial point of view.— *Organic Chemistry.* The derivatives of carbon; elementary analysis; molecular constitution; the series of fats; the derivatives of carbonic acid; sugars; fermentation; the aromatic series; colouring matters.

CHEMICAL AND PHYSICAL LABORATORY. (Three hours).–Measurement. Use of instruments. Physical problems. Chemical manipulations.

Add to this that, in all the sections one hour a week has been devoted during the last year before entrance into University.

COSMOGRAPHY. The earth. Universal attraction, (Newton's Law of Gravitation). The fixed stars. The diurnal motion. The annual motion. The sun. The moon. The planets. The comets. Shooting stars. Nebulae and the Milky way.

Thus by the study of Cosmography the spatial conceptions of the pupil are exercised in the course of it and he learns to make use of the technical information he has obtained in the study of Geometry and Trigonometry.

Geneva University is not a great university in the continental sense though it has, and always has had, men of the first rank among its professors. It will be seen however that the knowledge which it requires from those of its pupils who enter it from the College de Geneve is far in excess of that required by any university of the British Empire. I believe however that little by little this will be put right by us, and I should like to think that the great University of Calcutta would be the first to show the way to the universities in other parts of the Empire.

The democratic character of the Swiss institutions, and especially those of Geneva, where the older Genevese aristocratic families are absolutely without political influence, is sufficient guarantee that in its programmes, it is not the intellectual preeminence of a few which is aimed at, but a satisfactory education of the general mass of ordinary citizens, constituting the cultivated classes. This has been explained to me by the Genevese themselves. They even go so far as to defend their system on the ground that the clever boy will make his way in any case, whereas the boy who is not exceptionally clever will fail utterly if he is given too hard a task.

It should be remarked that, in point of fact the University of Geneva is embarassed by a large number of insufficiently and badly prepared Russian students as well as various other students from the Balkan states whose knowledge of French is not all that could be desired, and that it has found it necessary of recent years to confine its attention as far as mathematics is concerned to courses suitable to such students, and other persons whose main subject is Experimental Science. The best mathematical students from the College de Geneva now go to other universities in Switzerland and elsewhere, often to Paris, instead of remaining at Geneva. To understand the nature of the university course which the preparation at the College fits its students for, we must necessarily therefore turn our eyes elsewhere, either to Paris itself, or to Germany, from which the Sw iss universities in question, especially Zurich and

Bale, draw their inspiration. The fact that at this point we leave Geneva, is not to be taken as implying anything like want of recognition of the civilising role. Switzerland has, through Geneva, played in the East, nor of the importance of the original work of the eminent scientists who have taught there, in that Geneva was regarded by Ruskin as the heart of Europe.

The University course proper, Post Inter. Arts and Inter. Science courses, B.A. and B.Sc. Examinations.

It is only after the knowledge has been obtained which corresponds to the successful negotiation of the Inter. Arts and Science Examinations as defined by the existing schedules of Calcutta University that the university course in a continental sense can be said to have begun, or to be in any way possible. In the reorganisation of the University of Calcutta in such a manner as to render it a great teaching university, worthy of taking rank with the universities of Europe, attention must be particularly directed on these next two years. If we follow the German plan, the students will not be haunted by the fear of examinations at the end of them, and will be able gradually to acquire new intellectual senses, and learn to adapt their minds to entirely new processes. They will however from the first be aware that their aim, if a high one, will involve the creation of something new, extending the bounds of human science, and if more modest, will at least include the writing of a simple thesis which will summarise and throw light on results which are already individually known. If on the other hand the French system be adopted, the knowledge that each of the certificates which the students have to get may be got separately and at his leisure, and that they can only be obtained by real assimilation of modern ideas, and of modern progress, due to living writers among others, will mark in the mind of each student such at a distinct break in his habits of mind and the point of view from which he regards his subject, that he will be prepared for teaching of quite a different kind from the professor or lecturer than that to which he has listened hitherto. Teaching which before this period may have been not only adequate but admirably adapted for its purpose would do only harm if continued. From now onwards there should be ever increasing reference to the existing bounds of the science, and the steps being taken to extend them. The student should begin to know the names of the great living exponents of his subject, and should always from the first be encouraged, nay, find it necessary to consult from time to come original memoirs, the time so spent increasing of course as the months go by.

The alternative to turning to French or German models, as practically all civilised nations have done is to ask what Oxford and Cambridge are doing. It is unnecessary to refer to the other universities of the British Empire in this connexion, for they are all without exception content to have their best pupils, after taking their degrees with them, go to Oxford or Cambridge to recommence from the beginning the work of a freshman. I am sorry to say that some of their most influential heads and teachers openly regard these institutions as preparatory to Oxford or Cambridge, or as only suitable to the preparation of an inferior class of students. Only the more enlightend have set before themselves the aim to try to transform the institutions with which they are connected into universities which could take rank with those of continental Europe.

London indeed, the prototype of Calcutta, once an examining university only, and now making, thanks to Sir William Ramsay and others, almost frantic efforts to approach a continental ideal as a teaching university, is still so inchoate that no human being can say what form it will ultimately take. It may be feared indeed that one consequence of the war may be to bring in a Conservative reaction in favour of things as they are in matters academic. Those discontented with things as they are may not find it possible to risk the charge of want of patriotism which an insistance on examples set elsewhere might bring down upon their heads.

Remain then Oxford and Cambridge. But even an Oxford mathematician becomes apologetic when he talks of Oxford mathematics, and an Oxford man who is not a mathematician will be the first to tell you to send your boy to Cambridge, if you want him to learn mathematics. Till lately Oxford had none the less the great advantage of allowing its student plenty of time. He was not hurried or worried, and consequently could find time, if it was suggested to him to do so, to consult original papers in French and German even. But later changes have been introduced to assimilate the Oxford system somewhat more to that in vogue at Cambridge. In any case for the purpose in hand, any one who knows both universities would at once agree that it is sufficient to take Cambridge.

The Cambridge system has been recently modified. I am glad to say that I myself was one of a committee of seven, chiefly instrumental in accordance with the instructions given to us, in the destruction of the system in vogue before 1905. How unsatisfactory this system was, may be gathered by a single utterance made publicly by a man who is now Professor of Mathematics in the University of Cambridge. He said that at the end of the complete course of nearly four years, the best mathematical students not only were not acquainted with modern work, but were actually unfitted by their training from undertaking any original work of their own whatever.

The new system then carried did not at once come into use and it has not been working long enough to be judged on its merits. Moreover the great majority of university and college lecturers actually teaching still date from a time anterior to discussion, even urge on the need for change. They are quite incapable, for the most part and it is no discredit to them to say so, to properly administer the new system of studies and examinations which have been substituted for that under which they themselves grew up mentally.

America, the country of the Future, and Russia the Great Unknown, need indeed special consideration. The former country followed at first the traditions of Oxford and Cambridge, and to a certain extent those of the Scotch universities. But it has now for some time gone direct to France and Germany for its inspiration, though it has not yet succeeded in quite raising itself to their level as regards the quality of its scientific production and the excellence of its academic training. In one respect however it is already raced far ahead of the Old World, namely in its libraries and equipment generally. So much is this the case that, to equip a new university, in India or elsewhere, both as regards libraries and buildings, as well as other accessories, without reference to what has been done in America would be an act of great unwisdom.

Russia has recently made an extraordinary spurt in almost all the branches of knowledge with which I am superficially acquainted. Work which countse is now being done frequently by comparatively young investigators. These men are in almost every case, men who have studied in France or Germany or in both countries and have drawn their inspiration from what they have seen and heard there. In the rare cases where such a residence has not taken place, the work done forms a continuation of or is based on, work done by scientists outside Russia.

The Russian system itself has been marked by peculiarities which have at once served to exhaust the energy of the professor and to discourage even the patient effort of the Slav, illuminated though it often, is by the flash of genius. Efforts have been and are being made to alter it, but the lesson her as elsewhere is the same; however accidental the causes may have been, which may have led to the elaboration of a particular scheme, however badly executed it may be, as soon as it is established it is not the circumstances which led to it, but the fact that it is there which counts. Stupendous efforts made at a later date will scarcely avail to alter the result of a snatch decision of a committee, gained by the doubtful casting vote of the chairman. This fact should at once render us not unduly hostile to a proposed reform, and particularly careful to see that such a reform, if carried through, is carefully and consistently elaborated.

We have so far considered the College Matriculation and the University Matriculation Examinations (that is the present Matriculation and Intermediate Examinations) almost if not entirely as regards their direct bearing on mathematical study. It does not come within the scope of the present Report to discuss what other subjects form part of a general education up to this point. But some reference is unavoidable, to modern languages. It is essential now a day for any investigator to be acquainted with more than one European language, and he must ultimately be more or less well acquainted with four of them, English, French, German and Italian. The question arises at what stage the Indian student should commence the study of his second European language, and on considering this question we have to bear in mind that he does not in general study either Latin or Greek. In certain other respects also his case may be regarded to that of a student of a small country such as Denmark. The Danes have found that they have no time for Latin and Greek. Their own language does not count for scientific or commercial purposes. In certain respects the Danish student starts therefore one language behind his English, French, German or Italian rival or colleague. He replaces accordingly Latin and Greek by English, German and French.

It will be sufficient however to consider the plan adopted at the College de Genéve, omitting from consideration the Classical Section. In the semi-classical or Real Section, besides Latin, English, French and German are all studied to the end of the course, while Italian is voluntary. What it is important to know, is when these languages are begun. We have not to ask when the first of these is begun, this is of course French, the maternal tongue, and corresponds to the vernacular for the Bengali student. German is also begun in the primary school just as English is begun in the Indian primary school. The third language with the Genevese is English. This is begun two years before the period which corresponds to the College Matriculation (actually the present Matriculation Examination) in the scheme which I propose. Thus in the ideal system in India some knowledge of French should have been supplied at the existing secondary schools, and some very easy questions should have been set on it in the examination which corresponds to the present Matriculation.

It may however be argued that though this result is achieved in Geneva by students who carry the additional burden of Latin besides their own vernacular and another language, the conditions in England are such as to render it impossible there. But even if we accept this view, it is clear that French ought to be a compulsory subject in the examination corresponding to the present Intermediate Science or Arts, if the same results are to be obtained in Calcutta as in Geneva by the averagely intelligent pupil.

The third foreign language for the Genevese is Italian. Instruction on this subject is entirely voluntary, and may be taken during the last two years only of the school course. This would correspond in Calcutta to either German or Italian being taken as a voluntary subject, should the student choose to do so, during the two years preceding the present Intermediate Arts or Science examination (University Matriculation Examination, in the proposed scheme).

Proposals with regard to the General Organisation of the Mathematical studies in the University and what is regarded as the proper commencement, namely the period subsequent to the passing of the present Intermediate Examination.

Besides the ordinary courses of lectures to which reference will be made further on, the following innovations require to be introduced :

(i) A PRO-SEMINAR for the younger students, specially those in their first- year. In these, examples are to be done in class, partly by the professor, partly by his assistant, and partly by the students themselves. The subjects in which these examples are required shouldi nclude the Infinitesimal Calculus, including elementary Differential equations, the Theory of Series, and Definite Integrals, including the use of the Complex Variable and Contour Integration :

(ii) A SEMINAR in which special subjects are to be studied and lectures to be given by the students themselves in what they have been reading and a careful type-written summary should be preserved of each days proceedings. The Professor or Professors criticises and from time to time elucidates points which arise.

(iii) A COLLOQUIUM is intended for persons who are either already Doctors, or teachers of Mathematics or in certain cases preparing for their Doctor's degree. At the Colloquium, current mathematical literature and results obtained by the members, are discussed and short lectures given.

Besides these three institutions, closely related ones in the subjects allied to and closely connected with Mathematics are desirable. In default of these advanced students in one or other of the Physical or Natural Sciences may be asked to attend the Seminar or Colloquium, with the object of explaining any points which have arisen in their studies which appear to them to require mathematical treatment.

Similar remarks apply in respect to the connection between actuarial work to Mathematics, and the relation of the latter subject to Political Economy. As regards courses, besides the ordinary ones, one on Descriptive Geometry, accompanied by a Drawing Class will be required.

Lectures on the History of Mathematical Ideas, and the bearing this has had on Philosophy, should, if possible be entrusted to members of the Philosophical Faculty, who are mathematicians as well as philosophers. In default, the History of Mathematical Ideas may be dealt with by the mathematicians, and the connection with Philosophy and the philosophers. It should not be difficult to keep a close connection between mathematicians and philosophers, more especially if the philosophers themselves hold a Seminar or Colloquium, or both and invitations are extended on suitable occasions by the philosophers to the mathematicians or vice versa.

We now come to the courses which will be required during the first two years following the Intermediate Arts and Science Examinations. These in our scheme correspond to the first two years of University life in France, Germany, Italy, and Western Europe generally. Regarded from a mathematical point of view, they should evidently be spent in much the same way as they are spent in those countries. There are clearly three main divisions of mathematical studies to be considered in this connection: Analysis, Geometry and Mechanics.

To take first Geometry. Italy has, by universal consent obtained the right to be arbitor in this field. In France mathematicians are pretty much at the point where they were left by the great treatise of Darboux, which is concerned chiefly with Differential Geometry, and little else is there studied, though Picard, in a book published in collaboration with his pupil Simart, has expounded some of the results of the Italian School. In Italy, on the other hand, to the work of Bianchi which resembles and carries on to a higher degree of development and on original lines the work of Darboux, must be added an entirely different school, of whom the great living exponents are among others Bertini, Segre, Castelnuovo, Enriques and Severi. No English book exists which is suited either as a preparation for researches in either of these two kinds of Geometry. Forsyth's Lectures on Differential Geometry do not attempt to cover the necessary ground, and in the other directions there is nothing. We must clearly turn to the Italians themselves if we are to learn how best to prepare future geometers for the first two years of their university life. Any one acquainted with modern Italian Geometry recognises at once that the works of Salmon, admirable as they were at the time they were written, are quite unsuited for the purpose. The Italians advise, and actually employ, the Lezione di Geometria analytica, of Castelnuovo, of which the second edition was published in Rome in 1909, and in which the old Cartesian and the more modern projective and other synthetic methods are employed side by side. The

Italians allot also a certain place in the instruction of these two years to Discriptive and Projective Geometry, regarded as forming one separate discipline and sometimes as two. In either case actual drawing exercises accompany the instruction, and serve to illuminate it.

In Analysis there is a far wider range of choice. France, Germany, Italy and Belgium have all produced books of the first order of importance, and in England equally there are one or two recent books which deserve to be systematically studied. Of Italian books we may cite Dini's Lezioni di Analisi Infinitesimale (Pisa, 1907), and Capelli's Istituzioni -di Analisi algebrica(Naples, 1902). Among French books that by the Frenchman Goursat, Cours, d' Analyse of the Belgian de la Vallee Pussin, are both notable, the former as not only illustrating the modern tendencies, but also as embodying the course officially required for the French Licentiate, while the latter is remarkable from the fact that the author contrives in a small number of pages to give a lucid and inspiring account of many of the very latest results in the domain of the Calculus, Differential and Integral alike. While there is no work in German which reaches anything like the standard of these works, mention may be made of Burkhardt's volumes on Algebraic and Infinitesimal Analysis. This does not indeed pretend to do more than to introduce the student to some of the ideas at the foundation of Analysis. Among English books that by Bromwich on the Theory of Infinite Series and by W. H. Young on the Fundamental Theorems of the Differential Calculus stand alone in their atempts to give an account of modern research.

In Rational Mechanics on the other hand the English reader does not need to turn to foreign treatises, though he will almost certainly gain by doing so. The German treatment is more philosophical, and French writers are as usual elegant and at times profound. But for the purpose of the ordinary English-speaking student, the works of James and of Routh on Mechanics of Rigid systems and Love on the Theory of Elasticity will probably suffice.

Considering these branches of Mathematics in detail, and returning to Geometry, I may say that my own personal experience has shown me that the English training in Geometry, which has prevailed with various attempts as modification for the last thirty or forty years, is a complete failure. In the whole of my mathematical acquaintance, I know only three men in England whose knowledge and grasp of geometrical ideas is worth considering at all. Of these one has devoted himself ever since I first knew him more or less continuously to the study of Clebsch, and another more recently has attempted to make himself acquainted with the work of the Italian school. On the other hand, I have known a Englishman of considerable reputation among English mathematicians taking a dozen pages to verify analytically a question which I had set, of which the proper solution required not half as many lines.

In Analysis on the other hand all the books in common use in England are of a character which cannot be paralleled in current continental literature. Differential and Integral Calculus are treated almost entirely empirically, except in the case of the single book quoted above, and not even an attempt is made to deal with the difficulties which arise, still less to give a modern and philosophical treatment. It thus happens that a student brought up on such textbooks is at the end of his course in some respects worse off than if he had not gone through the course at all ; he has at best learned to work out a number of examples, and apply a certain number of rules, a result which may be of use to him if he is to be an engineer, but which will be worse than useless of itself, if he aspires to be a mathematician.

In Mechanics a quite unnecessary amount of time has been commonly spent in problems of an artificial nature in England on Statics and Dynamics of a Particle. A student who knows nothing of Mechanics at his entrance into the University, except what he may have acquired from a brief course at school, accompanying one on Experimental Physics, should be able in a year after his entrance into the University to already possess a sufficient mastery of Lagrangian equations in Rigid Dynamics, and understand the nature of the difficulties which arise in the problem of three bodies. An exceptionally intelligent student may be expected to do more even than this, and to have already gained some knowledge of Elasticity and Hydrodynamics.

Graphical Statics will of course form an important part of that portion of the course which deals with the equilibrium of bodies.

An essential part of the preparation for the mathematical study of the physical sciences is the Mathematical Theory of Probability, and I am of opinion that a course on this subject should be compulsory to students of Physics, Chemistry and Philosophy, as well as for the mathematicians, properly so called. It is also desirable in the course on Mechanics, which has too frequently been concerned only with the movements of solids and liquids, that a detailed mathematical treatment of Gases should also find a place, the obvious book whose study should be required in this connection is Boltzmann's Gastheorie. Perrins modest but remarkable little volume "Les Atomes", preferably in the original, may also be studied with advantage. I texemplifies the growing use by physicists of mathematical reasoning.

On the pedagogic side, if for no other reason, elementary courses dealing with the Foundations of Geometry and the Arithmetical Concept of Number, the former at least treated historically, and specially suited to intending secondary school teachers, might well find a place, among those lectures to which attendance is invited in the earlier years of the university course. Some students may however prefer to put off these subjects till they can study in greater detail the disciplines to which they naturally lead, namely Non-Euclidian Geometry and the Theory of Sets of Points. On the other hand, some slight acquaintance with these subjects will be found an advantage by the more brilliant students at quite an early stage of their career. It is clear that no general rule can be given in view of the varying limitations imposed by considerations of time and space.

It is clear also that a somewhat different mode of treatment to that most suitable for future mathematicians may be desirable for students who have already decided that their chief interest lies in one or other of the experimental sciences.

But it is essential that even such students should be introduced in the very first year of the University course, properly so called, to mathematical ideas, and not merely exercised in mathematical examples of a more or less trivial character. Some facility in dealing with symbols, and some power of reasoning about particular geometrical figures is of course necessary, but this must not be purchased at too heavy a cost of time and energy. Of the two extremes it is better that examples should be entirely neglected, rather than that too much attention should be paid to them. Indeed the professional mathematician is bound to have time during his long career to acquire a facility in those processes which he has occasion to employ, and the same may almost be said of the scientist who studies mathematics merely as a means and not an end.

With regard to the difficulties of language, which though intrinsically only of the second order, loom large to some, it may be pointed out that in the first two years, if the lectures are suitable and based on the books recommended above, the student will not need at most to do more than occasionally to refer to them himself. One to whom such consultation presents no difficulty will of course have an advantage as compared to his less well-equipped fellow. But this is inevitable and in some respects a thing to be desired rather than avoided. On the other hand, the university staff may well be asked to consider the question whether it would not be worth while to write books to be used in the University, written on more or less the same lines as those above recommended in French, German and Italian. In the early stages of the development of the Mathematical Department, labour spent in this direction might be well-spent.

The character of the examination will of course, if these proposals are accepted, need modification. There is no reason why this should entail an increased burden on the student, at any rate after the first year or two. The mathematical world in Calcutta will no doubt from the very first, find some slight difficulty in adapting itself to changes which may seem to it to be somewhat radical, touching as they do the B.A. and B.Sc. Examinations, in which so many of the affiliated colleges are interested.

Under the conditions which prevail in India, it is in my opinion absolutely indispensable that the whole of a definite building, or at least an important, large and carefully chosen part of one isolated from the rest, should be set aside for students of Mathematics in connection with the University of Calcutta. It is absolutely essential that all the advanced treatises and periodicals should be accessible to them in this building. It is undesirable unless in very exceptional cases that such works should be removed from the building.

It is highly desirable that this building should contain a room for Drawing, specially fitted up, so as to render it suitable for exercises in Descriptive Geometry.

As regards the extent of the library, and the size of the Reading-room, these must necessarily be planned on a generous scale. All the mathematical periodicals, as well as the Proceedings of the Royal Societies which contain Mathematical papers, must necessarily find a place there, as well as works on those portions of the allied subjects in which Mathematics is employed. The Mathematical Institute so created would be open to specially qualified prosecuters of researches in these allied subjects in so far as their work is mathematical. It would thus serve to bring together, not only mathematicians, Pure and Applied, equally whether they were professors, or schoolmasters, or candidates for the degree of doctor, it would also form a rendezvous for actuaries, mineralogists, physicists, political economists, statisticians, astronomers, chemists, geologists, zoologists, botanists, who have occasion to employ, mathematics, and to take counsel of mathematicians.

The Seminars and the Colloquia would naturally be held in the building as well as the meetings of the Faculty of Mathematics. The building would contain a room for the professors, preferably one for each, as the conditions of Indian life are such that it is not always possible for the professor to see the students at his own house.

It is absolutely essential that the building should occupy such a position in the city that the rooms are all of them quiet, well-aired, well-lighted, free from disturbance of every kind.

All the reasons, in fact, which exist in many European countries and in America for the existence of such buildings have tenfold force in India, and especially in Calcutta.

Bearing in mind that, wherever in the advance of human learning, the study of the Quantitative has begun to exist side by side with, or even to supplant that of the merely Quantitative, the aid of Mathematical Science is absolutely indispensable and bound to be recognised as such more and more as the years go on. It is impossible to conceive of a more useful commencement to the task of creating a teaching university than that of endowing it with a building such as I have in mind, exclusively for the use for the employers of Mathematical Science, and its adepts, together with the aspirants to mathematical culture. Indeed it is not too much to say that, without such a building or its equivalent, progress is likely to be imperceptible for years to come in Calcutta.

It is unnecessary to insist on the fact that there should also be a collection of models, and a small room devoted to the protocolls and seminars and colloquia. All the advanced courses would also be given by the professors in their building. The general mathematical lectures and those of an elementary nature, given to those students who have just passed the Intermediate Examination (at present so called), would not necessarily, and perhaps not naturally, be held in this building.

One of the lecture rooms at least should be so arranged that it is possible to a Magic Lantern, and the arrangements, with regard to blackboards and the writing up of formulae should be of the latest and most improved kind.

One or more type-writers should be provided and rooms in which then could be used. The services of a type-writer, and an official messenger should be at the disposal of at least the senior professor, who might be dignified with the additional title of Director of the Mathematical Institute. In the first instance the Director would naturally be a European, and during his absence his place would be taken by a Deputy. Subsequently the management of the Institute might be vested either in a single Indian professor, or in a small committee formed of the senior mathematical professors.

I should hope that distinguished judges, and others, whom the necessities of a great career have led away from their first love, will be occasionally found in the Reading-room of the Mathematical Institute, and that it will be possible also to hold informal gatherings of a social character, at which men might be present.

ACCOUNT OF THE MATHEMATICAL READING-ROOM AT JENA

In 1908 the University of Jena celebrated its 350th anniversary. On this occasion a new University Building was opened, in which a number of rooms had been set apart for a Mathematical Institute. The plan of this institute including the Mathematical Reading-room, had been made by Professor Gutzmer, then Professor of Mathematics at Jena, but in the meantime called to another University. The original of the Jena Reading-room is however that at Göttingen, and all such existing institutes really go back to the initiative of Professor Klein.

A plan of the Institute, with excellent photographs will be found in Professor Haussner's account of the Institute, which I here translate " The rooms which form the Mathematical Institute lie on the second story of the new University Building, whose front is shown in the first of the appended illustrations from the North-east. They occupy the eastern half of this story. Eleven of the windows in front belong to it, as well as the ornamental gable.

You go up the main staircase under the tower till you reach the second story, then the Mathematical Institute lies on your right. Immediately opposite the stairs, a few steps lead up to the height of the last row of seater in the Mathematical Lecture room (A), which can also be entered by another door, opening on to the vestibule leading to the remaining rooms of the Institute. The length of the Lecture-room is 11,20 metres, and its breadth 8,90 metres. The seats for the audience are arranged in eight tiers, so that each row lies ten centimetres higher than the one in front of it. It was impossible to make the rise from row to row greater than this, as the height of the room, determined by the facade, was only about $3^3/_4$ metres. Nevertheless the students in the last row can see the lecturers table and the blackboard without interference from the people in front. The excellent acoustic properties of the hall are also evidently partly caused by this arrangement of the seats. There are 94 seats, shutting up when not in use. Eighteen of these the front and back eight, and two in the third row, have no desk in front.

A stage is erected for the lecturer, to which to low steps lead up. This is determinated towards the audience by a table, 6,550 metres in length and 0,70 metre in width, on which a low desk can, if need be, be placed. There is a great advantage in avoiding in this way the usual desk, which often prevents the blackboard being seen, properly. With this arrangement the blackboard is free, and the long table is convenient for the lecturer to put things down wherever he may be standing, drawing utensils which he has been using, or models and instruments for measuring which he wishes co exhibit.

On the wall behind the table there are three pairs of blackboards, which can be let down and up. Under each is 5 shelves for chalk and sponge. The breadth of each blackboard is 1,592 metres and the height 1,520 metres. The middle pair of blackboards are plain, but the outside ones are ruled in squares of 5 centimetres to a side by means of faint shiny lines. This rich provision of blackboards not only enables the lecturer to avoid the tiresome and tedious rubbing out during class but also renders it possible to leave formulae or figures on one or two of the boards from one lecture to the next, even when the lecture-room is used between lectures. Over the central blackboard may be seen in the third illustration a sliding rule of 255 metres in length, made by Wichmann of Berlin. The scale of this sliding rule is eight times that of the small sliding rules made by the same firm, and used in the Institute by the students, the division of these being otherwise exactly like that of the large one. The sliding rule is hung up by means of a wire rope, working on pulleys, and can be let down to any convenient height by means of a reel on the right hand side, and fixed. On the left hand side of the blackbroads as you face them there stands a magic lantern (Epidiascope), made by Zeiss of Jena, and presented on the occasion of the opening of the new buildings by the firm. The apparatus is usually used for projections, and is therefore usually to set up that its object glass is at a distance of about 255 metres from the middle of the screen; the magnification is then nine times. The screen is two metres square, and is inclined to the wall at an angle of about 65°. When spread out it covers part of the neighbouring blackbroard ; it is therefore so arranged as to fold up, and is so seen in the third illustration. The screen consists of two frames, each a metre broad, hinged together, and covered with strong white paper; over this is white linen, which is only fixed to the outer frame, so that no break appears in the middle. The double covering prevents the passage of light.

Near to the magic lantern on the wall by the window in the switch for the electric light which belongs to the magic lantern. This light (30 Amp.) illuminates uniformly a field of 22 square centimetres. The town which provides electricity for the whole building, has in its wires a teosion of 220 Volts, whereas for the lamp including the resistance of the switch we only require 65 Volts. A resistance is therefore introduced under the switch by the window. During Mathematical lectures it will rarely happen that a number of figures are to be shown, one after another, what is important as occurring more often is these of the apparatus for a short time. Hence it is important for the lecturer to be able to darken the room quickly, noiselessly easily and without help. For this purpose it was found best to use an electric motor, which may be seen fixed on a bracket on the left hand side of the window. This effects the rolling up or unrolling of three black felt curtains, in wooden frames, in barely ten seconds. The switch of the electromotor is just on the right of that for the lamp, while on the left of the latter is a lamp for a control. Underneath there is the lever for guiding the electromotor backwards and forwards. The really excellent blackboards and the whole arrangement for darkening the room were made by the firm Max Kohl in Chemnitz. In the corner on the right of the window, under the screen, water is laid on, and from this the waterchamber of the magic lantern, when this is used for a long period, may be constantly supplied with running water.

Now although it is in general desirable to let the figures grow up under the eyes of the audience, yet the magic lantern is of great use, as it permits of our exhibiting figures whose production by hand on the blackboard would be impossible, or at least, would take up too much time. Thus for instance one can project the figures in the second volume of my little book on Descriptive Geometry. Descriptive Geometry (Haussner, Darstellende Geometrie. Leipzig, Göschen, 1908, pp. 112, 115, 117), referring to Dandelin's proof, and use these for explanation, while copies of the figure are distributed to the class. The great use of the magic lantern in Mathematics is that one can use figures in books or figures drawn in colours by one self when it is used as an episcope, whereas when used as a diascope, it is necessary to make a dispositive, which is expensive. Flat models too, as for example the models for the kinematic generation of cyclic curves invented by Professor Schilling, may conveniently and profitably be projected; if you turn the movable circle of the model in the Epidiascope, the generation for the curves can be easily followed on the screen.

The lighting of the Lecture-room is effected by means of four electric lamps (Sparbogen-lampen) from the Siemens and Schucke Works in Berlin. So as to obtain the requisite uniform distribution of the light, each of these is provided with a reflector for half indirect illumination, and they can be used in pairs. The blackboards are lighted up by 14 electric lights (Glühlampen) which are arranged on the ceiling parallel to the blackboards, and are shaded from the audience by shades of which three are visible in the illustration. The swithes for these lamps are next to the right hand blackboard, so that the lecturer is able at any instant to light up the board, which is very valuable on dull days. The electric bell G in the lecture-room,which is connected with the main clock strikes at the hours.

The remaining rooms of the Mathematical Institute are reached from the hall (C), which is cut off from the landing (B) by means of a latice-door. Each member of the Mathematical reading-room receives a key to this door, which is only to be opened without a key from the inside, and a second key is placed in the drawer of each desk in the reading-room. For these and other privileges the student has to deposit a small sum. The roomy hall (C), ten metres by 5,22, shown in the fourth illustration, contains five glass-cases for the collection of Mathematical models and apparatus, exception being made of those in use for teaching Descriptive Geometry, which are kept in the drawing class. The utility of the collection of models is much chanced by the fact that under or besides each model a description of the same containing all that is essential is attached in such a way as to be easily readable from outside. Even the larger apparatus, the Integraph (System of Abdank-Abakanowicz) and the large harmonic Analysor with five integration-apparatus, both from the celebrated factory of G. Coradi in Zürich, are kept under glass-cases to keep off the dust.

It was originally intended that the collection of models should be kept in the two rooms marked (D) and (E), which were planned as a single room. As however the reading-room (F) did not appear to be sufficient, this room had to be divided into two, and the room (E) was used as an annex to the Reading-room, while the room (D) was appropriated to the Director of the Institute. The letter room is therefore connected with the telephone L which is laid on all over the building, and contains another bell (K) striking at the first quarter after every hour ; a similar bell is in the hall (C).

The library is well provided with books and periodicals; of these the text-books and monographs are arranged alphabetically in the large Reading-room (F). The Encyclopaediac and Dictionaries are placed separately in the same room. In the little room, which is assigned specially to those students who are engaged in original work, there are the collected works of the great Mathematicians, as well as the periodicals; moreover the current numbers of a number of Mathematical and physical periodicals are laid out in this room. Here also are to be found the alphabetic catalogue, and the card-catalogue, which is in the making; the latter has been found to be really necessary, particularly so as to render the use of the library easier for the younger students. All these rooms are lighted by electricity (Glühlampen).

Next to the Reading-room in the Drawing Class (H), which can in case of need also be used as a lecture-room. Here the members of the class, who take part in the practical work, have their definite places. The length of this room is 11,50 metres, and the breadth, not counting the bay, is 9,50 metres. In consequence the places furthest from the windows are insufficiently lighted by the day- light, from the windows on the east side of the room, and so a skylight.

ON THE STUDY OF MATHEMATICS AT ENGLISH UNIVERSITIES

IN the academic life of England the Universities of Oxford and Cambridge have played not only the chief but until quite lately almost the exclusive part. Durham University indeed, the sole one besides Oxford and Cambridge which is not of recent date, has never been anything but a faint imitation, and this is certain particulars only of Oxford and Cambridge. Recently provincial Universities have been founded in most of the chief towns of England. At some of these Universities several of the Professors are men of distinction and the brilliant student occasionally get considerable benefit from their instruction, more especially in the natural sciences. In Applied Mathematics there is Lamb at Manchester and Bragg at Heeds, as well as Bryan at Bangor. But broadly speaking we may say there is no advanced school of Mathematics in England, in either Pure or Applied outside Oxford and Cambridge. The only attempt at a treatment of higher themes at all is to be found at Liverpool. In all these Universities without exception, the standard of Mathematics at Matriculation is extremely low, resembling that in the Little Go at Cambridge, differing only in this that optional papers are of some, what greater difficulty and range are permitted. It is not too much to say that in no part of the civilised world as far as I am acquainted, whether is the standard quite so low as in the Entrance examinations of the English Universities. I may say to London, the University of the capital, as well as to those of the other large towns, though a higher percentage has, it is understood, usually been insisted on there.

At Cambridge, and it will be sufficient far our purposes to take Cambridge and not Oxford, where the standard is somewhat lower, as is natural in view of the fact that Oxford has always shown a preference for Litterae Humanioris, even in the days when mathematics was its only rival, the standard at the Little Go is very much the same on paper at least as that of the present Matriculation Examination at the University of Calcutta. Practically however, all the best students, except those educated abroad, or privately, take examinations in lieu of the Little Go of a more difficult nature than this; moreover they pass these examinations two or three years before coming into residence.

The Little Go may now be regarded as the soft option for the weaker candidates, who are coming up to Cambridge for residential reasons, for the life of sport, pleasure and comparative leisure which it affords. Even these students, however, students by the way to whom there is practically no counterpart on the continent of Europe, frequently take the Little Go one year before coming up, or even sooner.

On the other hand, at the provincial Universities the students at entrance if of ability are often barely sixteen, and go on to Oxford or Cambridge after completing their course, thus recommencing their careers as Freshmen at the older University. The remaining students are commonly of inferior ability; they are destined for practical careers, or for the career of a schoolmaster in an inferior school.

The course of instruction at these provincial Universities is frequently such as to provoke a smile on the face of the continental man of science when he has occasion to refer to them.

It is only fair to say that these Universities have often been founded under great difficulties, and have owed their existence to the munificence of men possessing very little knowledge of what a University should mean, and that the level is steadily being raised as the Town Councillor becomes more enlightened, and the pressure of academic opinion more consistent. It must also be remarked that Mathematics is usually less satisfactorily treated than many other subjects. This is partly due to the fact that the city councillor sees the importance of research in the natural sciences, and partly to the fact that not only the engineer, but even the man of letters has been led in England to adopt a more or less hostile attitude towards Mathematical Science. The engineer has claimed that only the elementary parts of Mathematics were of use, and the philosopher has rightly objected to a subject, whose traditional treatment in England, in schools and Universities alike has been largely formal, involving at best practice in calculation, rather than an exercise in thinking, occupying the intellectual faculties and sometimes even deadening them, rather than illuminating the mind.

Things are however now changing in England. On the one hand the amount of Mathematics required by various types of intellectual workers and practical men has been constantly increasing, and on the other the gradual spread among the elite of at least a superficial acquaintance with what Pure Mathematics has come to mean in Germany, in France, in Italy and in Scandinavia, as well as in Holland, Russia and America, is fast engendering a respect for Mathematical Science on the part not only of the professed philosopher but even of the students of language.

At Cambridge as distinct from Oxford, Mathematics has for centuries been one of the most important branches of knowledge. Thanks to its institution of Senior Wrangler, it has loomed for a century larger in the public eye indeed than any of its rivals. The Senior Wrangler, as is well-known, used to be the man who came out top in the final examination in Pure and Applied Mathematics held at the end of three years and one term's residence. The name of this man was published in all the newspapers all over the country. There is a story on one occasion that a man who had in the morning just been declared Senior Wrangler at Cambridge, entered a London theatre; as he entered, everyone rose; whereupon he remarked in good faith to a friend who was with him; "I should not have thought they could have known the result so soon." In point of fact the Prince of Wales had entered the box immediately below that in which the speaker was standing. But the point of the story of course is that the man should be capable of attaching so much importance to the fact that he was first in this great examination of the Mathematical Tripos, as to suppose that the audience of the theatre rose to greet him. From this point of view, the story is been trovato, and if not true, might almost have been so. It is indeed really true that a comparatively recent Senior Wrangler on arriving home was entertained at a dinner given in his honour by the Mayor and Corporation. And it is said that he was met by a brass band at the railway station, and escorted from thence in triumph to the Town Hall.

For many years the Fellowships, then held for life, at Cambridge, were chiefly filled by men who had been Senior Wrangler or taken a place among the first few Wranglers. Most of these men never made any contribution to Mathematical science, indeed it was the exception to find one who had done so, while many soon forgot a great part of what they had learnt in preparin for the examination, and I have known one Head of a college to say that, with the exception of those in one or two of the earlier papers, he no longer understood the meaning of the questions set in the Tripos.

The question even in my time which was asked, when a name came up in conversation was never "What has he done?" but "Where was he?" this being short for "What was his place in the Tripos, Mathematical or other?" and practically all the residents could say off-head the place where any of their colleagues in one or other of the seventeen colleges which constitute the University of Cambridge, had taken in the Tripos.

In Mathematics accordingly each man had a certain number attached to him for his life, and I well remember one Wrangler telling another one in my Under-graduate days, "You will be known in future by such and such a number." The verdict of the examiners was an infallible one and was universally so regarded. One of the most distinguished of Cambridge men said to me on one occasion "A man as low as 11th Wrangler is no good at all", and it was difficult for the general mathematical public to believe that even the Wranglers after the first one or two, were not immeasurably inferior to the Senior Wrangler, who indeed not infrequently almost doubled their marks.

This system was later modified by the simplification of the examination, giving the name or Senior Wrangler, and the institution of a further examination following on this and of a more advanced character. This and the fact that a woman had already succeeded in beating the Senior Wrangler, with other events of a similar nature, had the happy effect of destroying the prestige of the Senior Wrangler to such an extent that it was found possible about ten years ago to abolish the order of merit, and so change the system that it is difficult to believe he can soon reappear. But lest any desire should be shown to resuscitate his elsewhere, and in particular at Calcutta, it may be well to repeat the statement already made in this Report, made by a present professor, that even the best students at the end of their University career at Cambridge were at the time in question, some 15 years ago, totally incapable by their knowledge and their training of undertaking any serious mathematical research. It may be remarked further that even an unusually satisfactory Senior Wrangler would at the corres--ponding period of his career in Germany have never been regarded as more than a promising young student, while his whole success in life would depend, not on what he had done, or on what he know, but on what he should do subsequently. Moreover his name would be absolutely unknown beyond the circle of his intimate friends in his own University. During the whole time I may add which I have spent in foreign Universities, I do not remember a single occasion on which anyone displayed the slightest interest as to the kind of credit a particular man had secured in passing his examinations. In the academic world of Arts and Science the sole question asked has been, what has he written, or what has he done?

The one thing at all corresponding to the institution of the Senior Wrangler is the coming out at the top of the Ecole Polytechnique at Paris. The man who does this has a brilliant future assured, but the examination is not in Mathematics alone, but in a number of Sciences, and it is far from certain that even in France the results of this competitive examination are good. The strain is certainly a very severe one, and many of the great qualities of an investigator are not tested by any examination, however skilfully administered. At the Ecole Polytechnique also the examinations are partly oral, as was the case, I believe, a century ago with the Cambridge Tripos. Again, owing to the very fact that the examination is not in Mathematics alone, it has not had the effects of a one-sided and stunting nature which appear to have been inherent to the Mathematical Tripos in its later days. From the age of 14 or 15 or even earlier, boys were not infrequently specially trained for the great Mathematical Tripos, subjects other than Mathematics were accordingly neglected, and only those parts of Mathematical Science being studied which paid in the examination. The latter baneful influence is also observable though to a less extent, in the Ecole Polytechnique, owing to the judicious manner in which its test are administered. At Cambridge moreover, a well-prepared student found it necessary to devote a considerable amount of time to practice in doing problems quickly and writing out bookwork in good style without a mistake, owing largely to the great length of the papers, whereas at Paris the time element has never been of the same importance, so that less energy has been wasted in these directions. As regards the scope of the Examination itself it has till quite lately been of such a nature that the most successful candidate is likely to remain in profound ignorance of mathematical discoveries nearly a century old. I well remember a man, who had taken high honours both at Cambridge and at a previous

University, devoting the leisure hours of several years subsequent to the time at which he was a Wrangler, to an attempt at obtaining the solution of the general quintic equation by means of radicals. It is possible that he had heard the name of Galois, but it is certain that he was totally unacquainted both with the Theory of Groups, to which Galois' work gave rise, as well as to the eloquently intuitive character of the proof it afforded of the impossibility of solving the general equation of any degree above the fourth by means of radicals.

On the other hand, the Mathematical Tripos has in more recent times, been of a nature to prepare a student in those parts of Applied Mathematics which are of use in Physics, at least in the ordinary treatment of this subject. Though the Theory of Functions of a Complex Variable only quite recently, about twenty years ago, introduced, has scarcely yet become part of the ordinary stock in trade of the English physicist.

In the present scheme, the Mathematical Tripos consists of two parts, the first of which is of a nature suitable for engineers and others whose interests are not mainly mathematical, but who are sufficiently intelligent to profit by a theoretical treatment of their respective branch sufficiently based on Mathematical reasoning. This examination can be taken almost at the beginning of the University course, so that the whole, or nearly the whole, or the two years and eight months before the B.A. degree, can be devoted by the able student to the advanced examination. This, like the M.A. and M.Sc. Examinations at Calcutta, consists of certain compulsory subjects and certain special subjects at the choice of the students. The students are divided into three classes on their work in the compulsory subjects, while an asterisk, signifying distinction or the reaching of a sufficient standard may be gained by a corresponding knowledge of the special subjects chosen by the student. In spite of the great inducement offered by the colleges in the shape of Scholarships and Fellowships, which can be enjoyed only while working for this examination and after successfully passing it, the number of candidates is far less than under the old Mathematical Tripos. This appears to be due in part at least to the inadequacy of the instruction provided at many of the colleges, the existing Lecturers having grown up under a different order of things. It does not seem to be due to a lack of positions open to successful students outside the University. There is in fact a dearth of candidates for such posts, particularly for the better Mathematical Masterships at the great public schools.

But this temporary eclipse of Mathematics or at least the diminution of the interest taken in it, is only temporary appears probable, if we enquire what is going on in other countries. In practical America, an enormous impulse has been given to the study of Mathematics in recent years by students returning to America from Germany and other parts of continent, its study is becoming more and more systematic all over the American continent, while it is becoming the rule and not the exception for the better class of student to aim at doing original work. On the other hand in Germany itself, where abstract thought has always been dominant, Mathematical Science was enjoying at the beginning of the present century what might almost be called its Golden Age in that country. And in what is now the Great University of Göttingen, there have been more than 200 students attending a single course of lectures of one or more of the professors, while the personnel of the teaching staff has had to be enormously strengthened. And what is still more striking, the leaders of industries have given a large sum to the University of Göttingen to be expended on the erection of a Mathematical Institute of a very complete character, a large building to be devoted solely for the purposes of Mathematical students and professors at Göttingen.

The diminution then in the interest taken in the subject of Mathematics at Cambridge as a branch of the higher learning, cannot be explained by the mere rivalry of the other sciences, but is to be explained on other grounds. Besides the character of the existing staff at Cambridge, which is such that there is a difficulty not only in securing suitable instruction and guidance for the students, and also renders the task of carrying out the examinations suitably, there is the nature of the examinations themselves, and a certain inevitable lack of coherence between the traditions of examining at Cambridge and the conditions involved in the new type of examination now instituted. Ever since the older Tripos examination was abolished in

1881, and a more advanced examination was introduced to follow the one which continued the tradition of the old one, there has been the great difficulty to overcome that the student had no sufficient guarantee that the questions asked in the examination would really correspond to the work he had done in preparation for it. It will be in my opinion still some years before this difficulty can be overcome at Cambridge. Indeed, doubt whether, without a complete change of system it will ever be completely overcome.

One of the great difficulties at Cambridge is exagerated importance still allotted to the examinations, owing to the fact that so many prizes continue to be awarded on the results of them. A clever student will naturally therefore avoid as far as possible an examination where he may fail to attain the highest measure of success owing to no fault of his own. He cannot afford to have to live down, perhaps till he is over forty, the fact that his success in it has been only partial, even if, which is rarely the case, he has private means, or other methods available, of replacing what he has lost at the outset of his career by such a bad start.

At the time of the earlier Tripos, an examination, known as the Smith's Prize Examination, entirely optional and only taken by the first two or three wranglers immediately after their Tripos, served to give the Senior Wrangler the possibility of stumbling, or of one of the other higher wranglers increasing his credit. When soon after the change of the Tripos examination in 1881, the Smith's

Prize Examination was replaced by an essay competition, each candidate being left entirely free to choose the subject on which he was to write. The results of this competition have been in my opinion on many occasions quite unsatisfactory. And it was bound to be so. The subject thought of by the candidate was not likely to be a good one if he trusted to his unaided efforts, and if he took advice of an older person the competence of this person was not likely to be always undoubted and, even when it was, the choice of the subject might still be unsatisfactory.

At the best the awarding of a prize for a piece of work done at the very commencement of a man's career can only be defended as a means of encouraging the first tentative efforts at research. But the consequent advantage to Science is completely overbalanced by the disadvantage of the first success, or failure, is to be recorded for or against the candidate at the subsequent stages of his career as is consonant to the tradition of Cambridge.

To understand how unsatisfactory this constantly keeping in mind the degree of academic success attained by a young man of science, it is sufficient to turn to the entirely different picture offered by the procedure in the selection of professors in Italy. The committee of the most famous professors in the subject in which the Chair is vacant, appointed by the Government ad hoc report on the various candidates, is only allowed to consider the work done by the candidates during the five years immediately preceding the election. And it is only in the case of candidates of world-wide reputation that work anterior to this period is even tacitly considered. The excellence of this procedure has secured for Italy a succession of brilliant professors, who more than hold their own, when the resources of the country are considered, with those of any other land in the world.

In England on the other hand a large proportion of the Chairs at the Chief Universities are held by men who at the time of their appointment would have been regarded on the continent as, at best, promising young students. These men have continued to hold the same posts for twenty years or more, and will continue to hold them till the time comes for their retirement, where such retirement is obligatory. Some of them will die in harness, and will, at the moment of their disappearance from the scene, still have nothing to point to but their original academic success at Cambridge. There are, of course, notable exceptions, but it is still unfortunately the case that the majority of professors and lecturers in the Universities and Colleges of England are men of this type. Indeed, as it not infrequently happens that they virtually choose their successors, or the successors of their colleagues, it is bound to take a long time before men who have a position in the science they profess, and are not mere teachers of the elementary portions of it, come to be exclusively elected, as is the case, almost without exception in all the great countries of the continent.

There is one other mathematical essay competition at Cambridge, open to all graduates of the University, known as the Adam's Prize. The subject of the essay is selected by a Board of Cambridge professors, and usually has some close connection with Astronomy or Theoretical Physics. The subject chosen is often one on which the professors know a particular man to be working, or to be likely to be willing to undertake. Here again therefore the effect of the prize is a mixed one. The winner of it gains in any case a reputation in Cambridge which almost of itself ensures a successful career.

At one or two of the Colleges, Fellowships are only awarded, this except at Trinity is a comparatively recent innovation, to men who have sent in an essay, or other piece of original work. The matter is however of purely domestic importance, except in so far as it illustrates the growing conviction at Cambridge that the production of original work is part of the duty of a mathematician.

At London, and at various provincial Universities, Doctor's degrees are obtainable for mathematics. But few such degrees are applied for, and the standard required in the original work sent in, is by no means always high. At London a single essay of moderate merit has, I am informed, been accepted. Elsewhere much depends on the referees appointed to report. On the whole, the possibility of obtaining such degrees has exerted hitherto an infinitesimal effect on the development of mathematical studies in the country. And it is natural that this should be so, if it be borne in mind that the courses of study in a provincial University are such as to leave an almost impossible gap between the attainments of a student completing his course and the knowledge and training required for doing a piece of original work of any value in mathematics.

It is perhaps well that this should be so, for the existence of this gap is one, which the friends of reform may well use as a weapon in the near future.

One difficulty which might not be suspected, is at the present moment a serious one, and has been such for many years. Men who have themselves produced something, and hold important positions, do not regard non-productivity on the part of their colleagues as a serious blemish. More than one man of distinction has been known to support indeed as candidate for a Chair a man of no scientific merit to the exclusion of a properly qualified candidate. In one case, the candidate so successfully supported was not even a member of a mathematical society. The excuse given, it is only an excuse, was that for the position in question no great gifts were required, as the work in mathematics at that particular University was of an elementary character. What is perhaps even worse is that in mathematics, as distinguished in this respect from other subjects, an unsatisfactory appointment of this kind arouses no single public voice of protest. An Australian delegate, who was visiting the Universities of Great Britain expressed to me not long since his amazement at some of the appointments made, and his strong feling that the Royal Society should take steps in the matter. Though there are private committees, called Watching Committees in other subjects than mathematics, not even with regard to these, however, has the Royal Society ever taken any sort of action, and they have uniformly allowed Fellows of the Society to be passed over, for men of inferior claims, without attention being called to this, even on the Council. It is in fact contended that as they have no locus standi on these bodies, their interference would be not only useless, but ill-advised. On the other hand, a possible action of Government is viewed with grave distrust by friend of scientific research. It is feared the only result of such interference would, in the existing state of the political world, lead to the cramping of research on its purely theoretical side, and thereby entrain a catastrophe.

The lesson to be drawn here as elsewhere is the same : the almost superhuman character of the effort required to amend an institution or a system which has been in existence for even a comparatively short time, and thus given rise to a progeny of vested interests, equally whether these interests involve patronage or power. At the same time we are encouraged for this very reason to ponder very seriously and to examine very minutely any new scheme which has been elaborated for the plans for any new institution which it is desired to set up. It is worthwhile to spend *now, days* of labour which may save our descendents *decennia* and perhaps centuries of fighting.

Though we have not specially alluded to the

Scotch Universities, and though they differ in many essential particulars from the English Universities, it is unnecessary to consider them separately. The differences are sometimes in their favour and sometimes against them, but for our present purpose these differences are negligible.

As regards the mode of selection of the professors, this is still more anomalous in Scotland than in England. Thus at one great Scottish University the electors at present for the majority of the Chairs are the Curators, seven in number, of whom four are tradesmen, or men of business in the town, one is a retired school master, one a Judge of Appeal, and one a former professor of the Medical School. The decision of these men is final, against it there is no appeal. The constitution of the Court of election is of a similar character in the other Scotch Universities. In England the electing bodies are of various kinds, but they are never a body of experts in the particular subject. No doubt occasionally every possible attempt is made to get the information as to the scientific capacities and position of the candidate, but the number of experts is few and those of them who are consulted, are not always the best qualified to give an opinion on the work of the candidate, nor is there, as far as I know in any single University, anything which remotely corresponds to the carefully reasoned out case which has to be presented to the Minister of Education by the Italian body of experts appointed to arrange the candidates in order of merit, nor anything at all approaching to the control which is thereby virtually exercised.

All the original. papers presented by the Italian candidates are considered by these experts and opinions of a detailed character expressed with regard to them. Even at Liverpool, where a Committee is appointed by the particular Faculty, and its report has to pass through Faculty, Senate and Council, a suggestion of mine inviting similar care in the presentment of the case was scouted by one of the principal professors on the ground that they had something better to do, and life was too short for such elaborate considerations. In practice at Liverpool the report of the Committee is final. The Senate has only two or three days in which to read the statement of the case, sometimes only a few hours, and it is expected, though not bound, to accept the report at a single sitting. Any motion to the contrary is likely to be viewed with grave disfavour by members of the particular Faculty and the working officials of the University. The Senate consists of the professors, who in this way are often unable to exert an effective control, the Council consists chiefly of town counsellors, to which has been added a few professors, six in fact in number, appointed for a term of years, one by the Senate, and one by each of the Faculties of Arts, Science, medicine, Law and Engineering.

As regards the original mathematical work produced under this system in Great Britain, it will be sufficient perhaps to state that, except for an occasional paper presented to the Royal Society, and a few usually of less importance published by the Cambridge Philosophical Society and the Royal

Society of Edinburgh, practically the only mode of publishing mathematical papers is by means of the London Mathematical Society or the Quarterly Journal of Pure and Applied Mathematics and the Messenger of Mathematics. The more important of these usually appear in the Proceedings of the London Mathematical Society, a periodical which has been so little appreciated in France, on account I am assured by one of my French friends, of its having been regarded as containing usually formal work of little interest, that it is not to be found in the Libraries of Provincial Universities, though by a tradition dating from the time when Cayley was Editor, the Quarterly Journal continues to be taken in.

The Proceedings of the London Mathematical Society are beginning to be appreciated abroad, but they are still far from having the rank of any of the Great Foreign Mathematical Journals.

It will be seen that, judged from a foreign standpoint, and by its intellectual output, the British School of Mathematics is not so successful as to suggest that Calcutta would do well to slavishly follow the system adopted by any one of the British Universities.

Moreover it must be borne in mind that, by universal assent England has done better on the whole in Applied Mathematics, exception being made of a few names than in Pure Mathematics, and that the genius of the Bengalee people, and of the Indian races generally, seems to lend them to interest themselves rather with abstractions than with attempts at the representation of the concrete.

Ireland, for the purposes of our report, may also be disregarded though it has one famous foundation, Trinity College, Dublin. The excellence of its bilateral general education acting on an imaginative, highly gifted nation, has given the Empire a number of distinguished writers, and among others, popular writers on Mathematics. George Salmon's works on Conic Sections, Higher Plane Curves, Solid Geometry are well-known on the Continent, and have been translated, with additions, by French and German writers. Moreover at a time when Cambridge was to all intents and purposes, ignorant of the original works of any writer but Euclid, Ireland has systematically encouraged the study of the great classical French-speaking mathematicians, above all Laplace. Unfortunately Dublin mathematicians have not made themselves equally well acquainted with the works of French and German mathematicians of the 19th and 20th centuries. Mathematical research in Ireland, in the sense in which the term is used on the Continent, may be scarcely said to exist at present, even in Trinity College, Dublin, the greatest of its foundations.

In intellectual matters the British colonies are not of course in advance of the mother country. The work of the early settlers did not allow time for book-reading, and it has only been by degrees that universities have been founded and endowed. Even among themselves these universities are on very different planes. In none of them the students engage in mathematical research, though some of their mathematical professors have not been unknown to fame. In isolated matters, India may have something to learn from some of them. Thus primary school teachers and others have to take a course at the University of Sydney, in the early history of Elementary Mathematics.

Canada has some advantages over Australia, put it still possesses very few universities, and I remember that in 1893 when I visited Quebec, a name known all over the world, with its historic associations, I found only one book-shop, and that only so called, because the facilities were there that afforded you for ordering books from Europe and elsewhere. Canada has been moving fast, and I have no doubt that even in Quebec, things have changed. But outside Europe, the only English-speaking communities from which the Empire can learn great lessons is the people of the United States, with its population drawn principally from the British islands, with contingents drawn from all the countries of Europe, but especially from Germany.

Mathematical research, like research in Physical and Natural Science, is rewarded by the fellowship of the Royal Society. The ettiquette is that the man should not himself ask any one to put him up, though this rule is, I am sorry to say, frequently broken, but that one of the existing fellows should write to a man who, in his opinion, is worth that distinction, asking his permission to put him up for candidature. The proposer then obtains the names of a number of other Fellows willing to support the candidate. Theoretically the Council as a whole elects, and there is sometimes warm discussion as to the merits of the different candidates. Practically however the Fellow on the Council, if there be one, who is regarded as representing Mathematical Science, pure or applied as the case may be, really selects the successful candidate. He may or may not ask the opinion of the people most likely to be able to judge the value of the work of the candidate. The result is that, while it is rare for a candidate whose work is of real value, not to become sooner or later a Fellow of the Society, it occasionally happens that one is elected who at the time of his candidature would not from the continental standpoint be suitable for election. It also happens that a candidate who is elected on a somewhat slender case, does no work subsequently to justify his admission. Possibly for these reasons, but also because 15 candidates in all are selected for admission to the Society each year, one of them usually for Pure Mathematics, and another for Applied Mathematics, the standard for admission into the Society is much lower than that for admission into the corresponding Societies abroad. There is no comparison, for example, between the distinction conferred by the Fellowship of the Royal Society and that of Membership of the Institute de France, of which the numbers are strictly and narrowly limited. The disadvantage of the French and Continental system generally is that the election of a man depends on the length of life of existing members. But it has the advantage of holding forth an incentive for the greater part of a man's Scientific activity. It is only by the strenuous work of long life that any but the exceptionally fortunate and very unusually endowed investigator, can hope to enter the Institute de France, and the same is true, though perhaps to a less degree, of the Royal Societies of Germany, Italy, Scandinavia, Holland and Belgium. On the other hand in England this, almost the final distinction open to a man of Science, may be earned at a very early age, for little more than a single piece of work, though some may have to wait, owing to special circumatances of their case, for as long a time as is common on the Continent.

The Royal Society itself rewards work with the award of Medals, Royal and others. These are virtually awarded by that member of the Council for the year who represents the particular subject, though this is not always the case. In practice the Council finds the work of awarding these medals arduous, and a doubt has been expressed officially by the Royal Society itself, as to the expediency of such prizes. Donors are in fact particularly requested not to leave money for such purposes. Speaking generally, it may be said that the fact that a man has obtained one of these medals, does not necessarily imply that the work he has done, is of a particularly high order from a continental point of view, while the fact that he has not got one, does not necessarily mean that in his own subject he is not one of the greatest living exponents.

The exponents of Applied Mathematical Science have been knighted by the King in recognition of their scientific eminence, and one, whose distinction was matched by the shrewdness with which he applied his science to practical life has received an earldom. But, I belive, no pure Mathematician has up to the present been so honoured. On the Continent, on the other hand, the Pure Mathematioian has probably received more recognition than his Applied colleague.

REPORT ON THE STUDY OF MATHEMATICS ON THE CONTINENT OF EUROPE

BY

Professor W. H. Young, Sc. D., F. R. S.

GERMANY & AUSTRIA.

Two important factors enter into the arrangement of a German university, essentially different from those underlying government of Oxford and Cambridge : (1) that the students are more or less impecunious and (2) that they are serious seekers after knowledge. At Oxford and Cambridge the first of these moments is habitually ignored, and has always been so as far as externals go. It has only so far been provided for that, more or less privately, so to speak, impecunious students are allowed allowances in the shape of scholarships and the like. The second of these moments has been till lately at least not taken into serious consideration at all. In so far as the provincial English and Welsh universities have provided far serious and poor students, it is not Oxford and Cambridge that they have turned for models, but to the Scotch universities which belonged at one time to the Continental system.

The serious student will not need the stimulus of competitive examinations to make him work, and accordingly the competitive examination has no *place* in continental university life. On the other hand, the choice of men for careers is considered in Germany as too serious a matter to be treated *by* the method of competitive examination, a method which enables inefficient and *lazy* officials to choose mechanically from among a mass of candidates a few for certain desired *posts*. In Germany such choices are made by painstaking experts, who regard each choice as a matter of importance to the country. On the other hand, the young men have been trained from their earliest childhood (9 years old), to regard every step in their school and university life as directly influencing their future careers; a false step is written down for ever on the book of their life, a book in Germany kept with fatal accuracy, and such a false. step cannot be retrieved by a brilliant success at the close of some period subsequently.

One consequence of the absence of the competitive examination is the almost entire absence of the cram-book. I say "almost", for unfortunately in practice there are faults in the system which in theory ought not to exist. One of these faults is the Abiturientenexamen, which though not a competitive examination, has certain of the deleterious effects of one, and certain others peculiar to itself. In chemistry, particularly, the teaching in the Gymnasia being still, I believe, very inadequate from a practical point of view, the cram-book exists in its worst form. This is not only true *or* the Gymnasium but also of the Realschule. I myself have had opportunities of verifying that at the

School of Medicine at Göttingen the men had frequently been spoilt for chemistry in this way ; few of them overcame the evils of their previous training. They learnt formulae by heart, and did their work in a wholly illogical manner, without using their intelligence. It would have been far better for them to have come fresh to the subject. In mathematics, however, the cram-book may be said not to exist in Germany.

The German student then may be supposed to come to the University burning to grasp his subject in its widest sense, prepared to devote all his time and energy to this aim, and to accept with gratitude any indications to be given him, by the wiser heads who govern his department, as to the means to be used in pursuit of knowledge. The Professor, on the other hand, feels that his duty is to put the students in contact with the master minds with whom he is himself conversant; he did not, twenty-five years ago, esteem it part of his duty to drill the students in the details of the various disciplines-this the student could be supposed able to do for himself; but the Professor felt that he must introduce the student to ideas, broad lines and methods, and draw him on, as soon as possible, to read for himself the original memoirs of the greatest thinkers.

For this purpose, books and periodicals had to be at the disposal of the students, and as the students were impecunious, it was no use to count on their buying even the most necessary for themselves. In this way, arose the excellent libraries, with their admirable arrangements, enabling the students to consult the books in a large and commodious reading room, or even, under certain restrictions to take them to their own rooms. The excellence of this plan was fully appreciated by the American students, and it is in America that the system of providing first rate libraries has since been most fully developed. In 1895, or thereabouts the G6ttingen Library adopted the American system of card catalogues for its University Library, and at that time an apprenticeship at an American Library was considered an asset in a candidate for a post at a German Library.

The University Library was however not considered sufficient for the needs of the mathematical students. Its disadvantages are two fold : firstly, the student has not direct access to the books, and secondly, the volume he needs may be out. Early in the 19th century, Seminar libraries were started at several of the German universities enabling the students to consult books at the university itself. In 1885 Professor Felix Klein started the first mathematical Reading Room at Leipzig, and in 1886 at Göttingen. But it was shortly before his going to America, (1893), Klein succeeded in incorporating the Seminar library with his Reading Room, and in inaugurating it on a larger and more efficient basis. The Mathematical Reading Room at that time consisted of a single large room in the second floor of the University building, close to the mathematical lecture rooms. The room was about seven metres by five, and had three large south windows and two similar ones on the east. There were four long tables running north and south, with drawers, and the students on payment of a small fee (five Marks for the semester of six months) could have each his own drawer with a key, and the appropriate place at the table. The walls of the room were lined with book shelves containing mathematical periodicals, memoirs and standard works, but few, if any, text-books. Very soon a second room was added on the north, a room which had formerly been the assembling and consulting room for the mathematical professors. In this second room, the current numbers of periodicals were neatly placed in pigeon holes, and the dissertations of the various Universities were arranged in chronological order, and properly indexed; there were also at first a few mathematical models in this room, but very soon the collection was removed to a neighbouring room, and very considerably enlarged.

It was one of the duties of the Professor's Assistant to look after the reading room, and he was supposed to be there at certain regular hours for this purpose. The students however had direct access to the books and periodicals and dissertations; they could look for themselves on the shelves, take any volumes they pleased to their places at the tables, and were bound to replace them properly before leaving the room. Under no circumstances was it permitted to take a book out of the reading room, except at the orders of the Professor during a class or colloquium, and then only to bring it to a neighbouring lecture room.

Silence was of course the rule of the Reading Room, but, at the period in question, the number of mathematical students was so small that one was generally alone in the reading room, except just before, or just after lecture hours, and not infrequently there were only two or three students present, and they would compare notes about their work. This was not possible later when the number of mathematical students had augmented. A few years later, there was hardly ever a vacant place In the Reading Room, and it had to be enlarged. It has, I believe, had to be enlarged again since then. Smoking was not allowed in the Reading Room. Water was laid on and there was a tap and glass there.

The funds for the maintenance of the Reading Room were supplied by the University, and came from the Prussian Government; only occasionally were books presented by the authors or publishers. Each student who subscribed possessed a pass 'key and could enter at any time when the University Building was open from 7 a.m. to 8 p.m. or even later. Three Marks was charged for this key. There was, after the enlargement of the premises, an anteroom with pegs for hats and coats, and stands for umbrellas.

The Reading Room was cleaned on Saturday afternoon, and at that time therefore was not available for works, though at a pinch one could go in to consult a book. Otherwise, with the exception of two or three days each semester the students could come when they liked in weekdays, and even on Sundays by the side door of the University, provided this was not locked.

The organisation of the Library and the Reading Room is such that they in many ways complement one another. Certain periodicals are taken in by the Library, and others by the Reading Room ; the rarer books are more often to be found at the Library, the choice of books in the Reading Room being such as is required by the average serious student, and not in general including books of great price. On the other hand, the organisation of the University Libraries throughout Germany, and certain other neighbouring countries, e.g., Switzerland, is such that books may on occasion, be sent from one University Library to another; if a book is required which is not in the University Library or the place in question, application may be made to the Head Librarian to get the book sent from some other Library, the contingent expenses being borne by the person requiring the book.

At the University Library, the student has not direct access to the book-shelves, except in the case of a certain number of books of reference (e.g. Die Fortschritte der Mathematik) which are to be found on the walls of the Reading Room at the University Library. The latter Reading Room is opened at stated times to all students of the university; books are to be applied for, by means of forms to be filled in from the catalogue. In all the best libraries card catalogues are used, as far the most convenient. Besides the reading room at the University Library there is the lending room; here the student, if vouched for, by one or two citizens of the town, may take out a certain number of books. The rules for the taking out of books vary in different libraries. One of the best arrangements with which I am acquainted is that in use at Geneva, where the book is given out with a pink slip in it having on it the day of taking out and the day the book has to be returned. A small fine is usually inflicted if the book is not returned in time. If not required by some one else, the book may be kept for a fresh period. It is advisable, I think, not to let the period' be too long, as not infrequently people take out books and do not use them, but keep them, thinking they are going to do so, in this way inconveniencing others. A good plan, also adopted in Germany and elsewhere is, that, if a book you require is out, the librarian writes to the person who has it, asking him, unless he is using it, to return it at once.

In the use of the libraries, the Professor serves as a guide for the student. He suggests to the student to undertake a certain course of study; for this, the student will probably require a certain treatise, or treatises, and it may be advisable for him, if possible, to buy one of these; if this is not possible, he will have to take it out of the University Library, or in the Mathematical Reading Room, as he will probably require to consult the treatise continually. On the other hand, his special branch of research will probably have to do with recent papers, and will start from these. It may be possible to get these sent gratis by the author, generally in the first instance the Professor lends them to the student. At a later stage the student will have to consult the original papers of older writers, some of which may perhaps be found in collected works at the University Library, while others are only to be found in the old numbers of the periodicals in which they originally appeared. From time to time the student reports his progress or brings his difficulties to the Professor, who is easily accessible in his consulting hour, (Sprechstunds) (generally immediately after his lecture hour), or with whom an interview may be specially arranged if the matter is a long one; such an interview generally takes place at the Professor's own house.

When the student's researches are sufficiently advanced, the professor invites him to give an account of them to himself and his class of advanced students. Such an account may take place eitber as one of a course of such discourses, given in turn by the students in the Colloquium, or it may take place less formally at an evening meeting of the Mathematical Club. The Colloquium is generally planned by the Professor in connexion with his own course of lectures, points of minor interest, or points requiring further elucidation, being chosen by him and given to the different students to work up and report on. The time given to prepare such a lecture would vary from a week to the greater part of the semester; those who undertook the first lectures at the colloquium would of course have to be already conversant with the matter, and it commonly happens that students who have already worked under the Professor for a semester or more, have already collected the material for these, while the newer students have the, later dates assigned to them, giving them time to work up the subject. It is usually in the shape of a subject for a report to the Colloquium that the students begin research. Sometimes the first such subject suggested by the Professor turns out so fruitful that the student may find it worth his while to continue his researches in that field, and so arrives at a dissertation for his Doctor's Degree, or subject for an article to be published in one of the Mathematical journals. More frequently this is not the case, but the practice is excellent, and after one or two such lectures have been given, whether by himself or his comrades, the students begins to have greater confidence, and knowledge of the mechanism of a lecture or a mathematical paper. At the same time the awe which he naturally at first feels for the great Mathematicians of the past, turns to a milder feeling of sympathy and fellow-feeling, while he learns to appreciate truly that in which the great ones of former times and of our own days have excelled.

Another good result of this mode of working is that several students are engaged on the same subject or branch of a subject, and they naturally discuss their work together. This yields a stimulus far greater than any other. In Mathematics as much as in any other subject isolation leads to narrowness and the exchange of ideas with equals tend even more to encourage individual thought than to sit at the feet of a great master. Of course the advice and control of the master cannot be dispensed with, otherwise too much time would be lost in doing over again things which have already been done, and in going up blind alleys.

The German Professors are alive to this fact and they have accordingly instituted at Göttingen and elsewhere a Mathematical Society of their own, meeting one evening in the week, to which the professor, privat docents and a few advanced students have access. The current Mathematical literature is as far as possible divided up among the members, for perusal, and subsequently to report to the Society as to the contents. Free criticism and suggestion is allowed, and in particular, any references to other writers ancient or modern in which the subjects treated of in the society occur are welcomed. In this way I have known more than once that it has been discovered in time that a piece of work supposed to be new was really already in print elswhere, and a stimulus has been given to the original work of a member by having his attention called by another member to a place in literature, current or old, which seemed to bear upon the subject. For instance, Professor Hilbert's work on Integral Equations was sifted and discussed over and over again in the Mathematical Society before it went into print, and a subject like the Riesz-Fischer Theorem, or the investigations of Weyl, Zermelo's Axiom, were in the air for some time before anything appeared in print.

The German Mathematical student coming from the Gymnasium at the age of about nineteen, does not by any means take the place of an advanced student. His knowledge of Mathematics is very embryonic, and hel has to work hard during three years for his State Examination, an examination in which there are a great number of subjects to be mastered, and which corresponds roughly to our B.A. Nevertheless, if he is a young man of ability, the Professor will encourage him from the first to attend his advanced lectures, and for this purpose he usually makes these fairly straightforward, not demanding much previous knowledge for the first two months. If after this, the younger students drop off, and find the lectures becoming too difficult, they will at least have had a glimpse of what their work is leading to, and gained in stimulus, if they have not grasped much knowledge. Some one or two will however find from the first that they are able to follow, and will find time beside their ordinary work for a certain amount of research. In the Statsexamen itself, a dissertation is required. The subject is generally of a critical or historical nature, as it is hardly to be expected that the student would at this stage be able to produce original matter. It is however a serious piece of work, requiring independent thought. The student is not treated like a school boy. Strict as the discipline is at the Gymnasium, the freer is the life at the University. "Frei ist der Bursch."

There is indeed another great difference between Oxford or Cambridge and a German University, this is constituted by what the Germans call "Lernfriheit." At the University the German boy, who has from his earliest childhood been subjected to the most rigid and uniform discipline, suddenly finds himself freed from all restraint. It is the golden time of the German's life; before it he is being put through the mill of the schools, no room is left in these for individuality; everything is cut and dried, everything is "vorgeschrieben" and must be performed in the "vorgeschriebene Weise." Afterwards he becomes the "Philistine," and, as such is again subjected to the strict discipline of German life, only to be relaxed in the sanctum or his own house, where he is the peaceable monarch of all he surveys, and exercises a mild sway, tempered with tobacco smoke. But at the University the German gets a glimpse of that forhidden land of liberty. Most of them have already most their initiative so far that they only so far enjoy their prerogative that they go to Kneipes, and drink unlimited beer. If they enter one of the corps, they are subjected to its rules, they have to do all sorbs- of things quite uncongenial, at least at first, such as going to see other people fight duels, and subsequently doing it themselves. I myself watched the hardening of a gay young boy who came up as a medical student, and nearly fainted the first time he came into the dissecting room. He took about half the first semester, but, being under the orders of his corp, he soon got all the squeamishness knocked out of him, and, as he must be now about 35, I make no doubt he has not turned a hair at any of the atrocities he may have been called upon to assist at in Belgium and France. Left to himself, that boy would have remained a gentle sensuous creature, fond of his dinner and his pipe, and selling groceries over a counter–for he would certainly never have passed his medical examinations. The rank and file of the German Universities, the men who correspond in some sort to the poll-men of Oxford and Cambridge, get shuffled into grooves by their more energetic fellow-students. What remains over is a small minority who really enjoy their freedom. Among these, there is a large proportion of Mathematical students in general, though this will also vary according to economic conditions. Roughly we may say, however, that the Mathematical students are more often serious in their work, and less often belong to Corps than happens in the other subjects. The Mathematical student, then, avails himself of the liberty accorded him to choose his own lectures, and to follow or not follow those he selects at his own will. This liberty would be merely nominal if there were not a great variety of lectures to choose from, but this is the case at most of the Universities, moreover there is the possibility of moving from one University to another, without losing any seniority.

It is not without importance to note the social rules at a German University. A student goes and calls on any Professor whose lectures he wishes to attend as a matter of course on the first, or one of the first, Sundays in the semester, calling, as is the custom, between eleven and twelve in the morning. He is received by the Professor en famille.

The relation becomes in this way a more personal one than in other countries. In former days, this was still more the case than at present, and in obscure Universities the old customs, no doubt, still linger on. The Herr and Frau Professor invited the students during the term to come and take a "simple bread and butter," as it was called at their house, and some of the most congenial of our student days in Germany where those simple family evenings, with their good plain black bread and pumpernickel and fine fresh butter and Herring Salad and slices of multitudinous sausages. Now these entertainments, if they still take place are less convivial. There is dancing, the party takes place at a Hotel, the simple food is replaced by more luxurious fare. Perhaps after the war the good old fashions will come back. The "simple bread and butter" was something really German (echt Deutsch), which ought not to die.

At these simple functions it was not considered ill-mannered to talk about one's mathematical work; there was no reproach in "talking shop", and it was one of the many occasions on which the students brightened their wits by rubbing them together, as well as by drawing nourishment from the fount of the Professor's erudition.

The organisation of a German University is very simple. Each Faculty has its Dean, who is one of the Professors of that Faculty, elected by it, but taking office practically in turn, and holding office for two years. At the head of the university stands the Pro-rector, also one of the Professors, and holding office for a year. The Deans perform the business of the several Faculties, and their work is very arduous; the duties of the Pro-rector, on the other hand are largely social, and the office is a serious expense to the holder of it. At the University the Government is represented by an official called the Curator, an office which, unlike that of a Dean or the Pro-rector, is a paid post. The office of Curator is sometime is of the nature of a retiring pension, but sometimes the Curator is a very active person, and, as he has much to do directly with the students, he is often a very popular character. All questions whether of University organisation, or of individual exceptional cases, which have to be referred to Government go to the Curator, who is, by reason of his office, quite independent of local opinion in the university. The final court of appeal is what is called familiarly the "green table", that is the Department of Education, the so-called "Culture Ministry", (Kultusministerium). In exceptional cases, as for instance on the occasion when for the first time a woman was allowed to go in for her Doctor's degree in Prussia, it may be necessary for the Curator to send the student in question up to the Ministerium itself. The persons there in authority are the Kultusminister himself, with whom the interview will be merely formal, chiefly marked by the fact that the Student has to say "Excellenz" at least once, and a number of competent assistants, Privy Counsellors (Geheimrate) experts on Educational matters, one of whom will be delegated by the Minister to listen to the affair, and to make the required decision, which is communicated through the Curator.

I see that Dr. R. V. Sterneck, in his report on mathematical teaching at the Austrian Universities claims that in Austria the principle, not only of "Lernfreiheit" but also "Lehrfreiheit" is carried through most systematically. The professors'—ordentliche "Professoren"-are state officials ("of the VIth class"). But they are only so far under orders as to what they are to do in the way of teaching that they are bound to lecture five hours a week in each Semester (at least) on the subject for which they were appointed, give a public and to lecture every third semester on some special part of that subject. The assistant professors-"ausserordentliche, Professoren"-are also State officials ("of the VIIth class, but only with a stipend corresponding to the VIIIth class", so that they are more honourable if less lucrative !). They are also bound to lecture five hours a week, but not to hold a public lecture.

The principle of "Lehrfreiheit" however is most fully exemplified at least in theory in the prcsons of the Privat dozents. They are not state officials, but University officials. They apply for the title which is accorded by the University under the sanction of the Kultusminister. They have no duties, and may lecture or not as they please. In Austria apparently this subject of their lectures is not only at their disposal, but it is regarded as unseemly of the professors to try to influence them, or to make them take up a subject considered by the professor in question to be convenient. In Germany this is certainly not the case. Göttingen the professors certainly dictated the subjects of the privatdozent's lectures, in so far as it was for their convenience or what they esteemed the needs of the department. In addition the privatdozents might lecture on special subjects chosen by themselves. I recollect perfectly when Zermelo applied to become a Privatdozent at Göttingen, Professor Klein saying that it would be convenient for the Faculty to have some one who had studied at Berlin, as he could be set to lecture on Variations, a subject which was needed by the Faculty. Theoretically there may be "Lehrfreiheit", but practically it only exists in a very limited degree, as is of course natural in a country so permeated with the military system as Germany. The good of the principle must not however be confounded with the evils which occur in practice, and it may very well be that in Austria. the system comes to its full rights. A propose of "Leahrfreiheit" I would take as an example the subjects chosen by Professor Klein for his lectures. About 1897, Professor Klein, who is professor of Pure Mathematics, took to lecturing on Mathematical Physics, although there was an eminent professor of Mathematical Physics, Professor Voigt at the university, who was not at all pleased at Professor Klein lecturing on his subject. The latter chose to do this because, he wanted to grasp the subject himself. For several years Professor Klein did not lecture on Pure Mathematics at all. Later he took for subject "The Mathematical En-cyclopaedia".

APPENDIX BY A FORMER STUDENT ON THE SEMINAR

The object of the lectures being to guide the students in their reading, that of the seminar is to encourage them in original work and research and in the art of composition and diction. At Göttingen there were two seminars in my time; one held by Professors Klein and Weber (Klein the active, Weber the sleeping partner); the other by Professor Voigt on Mathematical Physics. In the latter seminar I recollect doing a lecture on Helmholz's application of the complex variable to hydro–dynamics; and finding it extremely interesting; I do not however recollect that Professor Voigt did more than give me the subject, and talk to me sympathetically and with enthusiasm; I should not call him an inspired teacher, but rather an ardent worker, who expects independent work from his pupils; for this reason he was not popular with his class; more especially with the rank and file of it, and I was much surprised some years later to find he was considered one of the greatest attractions of Göttingen. I put this in by the way, as I have no particular recollections of Professor Voigt's seminar. As far as I recollect Klein and Weber were supposed to take alternate weeks at their seminar, and it is probable that others of the class, *e.g.*, M. J. prepared their lectures under Weber; as I did under Klein, I have myself no recollection of Weber at the

Seminar. The dominant spirit was Klein. During the first few weeks of the semester he gave me two subjects of research, (1) Schwarz's paper on the function, which he advised my beginning to study at once with a view to understanding his lectures on the Hypergeometric Function after X'mas,-he told me the paper was very difficult, and I found it so-and (2) Study's book on Spherical Trigonometry, which had just appeared, and which he also wanted to refer to in his lectures after X'mas ; as he himself had not read Study, he gave this to me and proposed that I should speak about it at the Seminar. This was Klein's constant practice, and he managed in this way to get a certain knowledge of a great deal more of mathematical literature than would otherwise be possible at the same time giving the student the invaluable stimulus of discussing the subject with himself. I very soon found a number of things in Study which required more than mere comment, and, after giving my Vortrag about February of that Semester, I continued my researches and my work was embodied in my inaugural dissertation printed in the following year. My companion, the American on the other hand was not so lucky; Klein gave some work of Bolza's or Bocher's on the Hypergeometric series to study, but was only able very partially to digest it; and the another Vortrag on it was uninteresting and led to no subsequent result. On the other hand Vortrag of, I think in the following semester, led to a little paper in the Math. Ann. containing a correction of a mistake in Riemann's classical paper; the matter was however considered by Klein too him for a dissertation, and afterwards wrote one on Lamés Differential Equations, I think, which both and everyone else found very wearisome was a little bitter about this all. M. and Dr. F. lectured at the seminar I remember on work which they had already done, and they were accordingly among the first in the semester to lecture at the seminar.

I read at home and then went to Klein's "Sprechetunde and had a little talk with him at the University. If there was any actual difficulty, he might be able to put me right at once, or suggest where I could look up authorities; in this latter respect Klein was invaluable, as he had a most remarkable memory for references and connexions. If it was a question of a longer discussion, he would appoint a day for me to come to his study at home "in the Wilhelm Weber Strasse at 5 o'clock in the evening, and I have constantly spent a couple of hours or more with him in this way; he was never chary of his time with his students.

The volume of Crelle containing Schwarz's paper I took out of the University library, and had it in my possession until Professor Klein required it himself about the end of January. Study's book Professor Klein lent me was with me until I bought it myself.

The following year, when Klein was lecturing on Differential Equation, he gave me as a subject Spherical Harmonics, illustrated by models which he had in the University collection and he also gave me Riemann-Hattendorf to read, and I believe I gave a lecture on Fourier Series, but I do not remember anything about this, except that the subject interested me, and I determined to return to it later.

In my last semester (fourth) I did not attend the Seminar, but I remember that a friend (I. M.) was given by Klein the subject of the Peano space filling curve, but I. M. did not do more than give an account of Peano's work, apparently not appreciating the full bearing of the subject, or see its possibilities.

A few years later, the plan of the Seminar was altered. By that time Professor Klein had given up looking after Doctors Dissertations altogether indeed I think I was the last in whom he took any great interest. Professor Hilbert took the dissertations more or less in the way Klein had done. The Seminar was utilised in a new way, more on the lines of the Physical Seminar in my time, as a means of getting the students to do examples; it was the duty of one of the junior professors to look through these examples. In Austria, I see from Sterneck, a Seminar of this. kind constantly exists simultaneously with the more advanced type, and is called a Proseminar.

The lecture at the seminar had not only to be given in public to the class, but an account of it had to be entered in the book kept for the purpose. These accounts form an interesting feature of the Göttingen, Lesezimmer and are valuable more especially for the very complete list of references which Professor Klein insisted on. Work of the type here referred to, though not suitable for publica-

-tion, is frequently of great value to subsequent investigators.

After, or even during the lecture at the Seminar, there was often a discussion, frequently of an animated nature. Professor Klein tolerated any expression of opinion from any member of the class, however humble, and even allowed himself to be taken to task in public by bolder members of the class. On the other hand he could be very sharp in his criticism, and was most conscientious in his conections both of the matter and the manner of the lecturer. We all learnt something of those tricks of the trade which made Klein such a good lecturer, and gave him such a command of the blackboard. What he could not tolerate was a long analytical demonstration, unless properly manipulated, with systematic use of the blackboard; he would first become restless, protest, and finally get up and cut the whole thing short, appealing to the class for a support which was seldom wanting.

The Göttingen Seminar in my time was, I think typical of this class of institution. The account of the Seminar in Sterneck agrees perfectly with my recollection. From that account I gather that in Austria the professor receives a certain part of the student's fees in remuneration for the holding of the Seminar, and that another part of them is set aside for small aids to the students, (40-60K), probably being used for the supply of books to the more eager of the class, I should suppose from the amount. In Austria each Seminar has its special room, with tables for work and a library :—the

Seminar library is perhaps the photo-type of the Göttingen, Lesezimmer. The number of hours assigned per week to the Seminar was one in Göttingen, but is 3 at Vienna, of which one hour is Proseminar, and varies from 3 to 4 in the other Austrian Universities.

The Seminar in Austria has certain statutes, granted by the Minister of Education, which permit the Professor, for instance, if he likes, to confine the membership of the Seminar, with the advantages which it confer in the use of the special room and library, to the active members, bound to hold a lecture from time to time as desired. This is however not usually insisted on, and the Seminar includes in general all Mathematical students who desire to form part of it. It is however in general obligatory to present yourself personally to the director of the Seminar, and get his permission to become a member, as well as to receive a key to the special room, and a list of the members is kept by the professor or his delegated assistant. These rules were as far as I recollect, also enforced at Göttingen.

There are people *(e.g.,* E. Bernheim, quoted by Sterneck) who regard the institution of the Seminar as the centre of gravity of the German University system. It has been proposed (Prof. V. Dantscher in Graz) that it should be made obligatory for teachers to have been an active member of a Seminar for at least 2 Semesters; this being regarded in the same light as the attendance at the Physical laboratory, and having to be vouched for in the same manner before the student is admitted to the examination.

State of Knowledge of the Students at the Time of entering the University

In Germany a student had in order to enter the University, to have passed till lately the so-called Reefeprüng or Abiturientenexamen. For this he requires in Mathematics Algebra up to and including equations of the second degree, plane and solid Geometry and plane (not spherical) Trigonometry. The elements of conic sections and the concept of coordinates. If he has not taken Greek but has taken Latin, he has in addition to go up to cubic equations in Algebra, to understand harmonic points- and lines, axes of similitude not in plane Geometry, and to have mastered the elements of Darstellende Geometrie; to have done spherical trigonometry, maxima and minima, and the analytic Geometry of the plane. If finally he has not taken either Latin or Greek, he is required to know the theory of series, and to be familiar with the most well-known series which occur in Mathematical analysis,. and to have mastered either the theory of equations with practical methods of approximation, or to have done further in his studies of Darstellende Geometry and synthetic and analytical Geometry of the plane and space.

It should be noticed that in no case is any knowledge of Differential and Integral Calculus required, and I have to add from my personal experience that the new Mathematical students were excessively ignorant of these subjects, and could not integrate the simplest expressions; it was for this reason, after seeing what happened at Cambridge, that Professor Klein sought to institute the practice of doing examples to be looked over by the Junior Professor above-mentioned.

In Switzerland, however, and presumably in France, the Mathematical knowledge demanded of the new student is of a more advanced nature.

EXAMINATIONS

There is only one proper University Examination for mathematical students; this is that for the Doctor's degree. The conditions are not quite the same in Germany and in Austria. In both countries you have to have been an immatriculated student of some recognised University for at least four years (in Austria, and I think in Germany also) ; years at foreign Universities are not always counted in full, but years at Oxford or Cambridge are given their full value. You have to present a dissertation, which has to be of a certain length, about thirty pages of proof at least, and to be recognised as proving that the author is capable of following research independently. If the dissertation has been accepted, an examination has to be undergone; in Germany this examination consists of a two hours examination on the principal subject, (*e.g.*, Mathematics), followed by an hour's examination in each of the accessory subjects, (*e.g.*, Physics and Astronomy). These examinations are oral; these examiners are the professors of the subject taken, and the examination takes place in the presence of the Faculty of Philosophy. In Austria there is only one accessory subject, and two principal subjects, in one of which the dissertation is written, while the two principal subjects are treated in an examination of two hours together, and the accessory subject in an examination of one hour. Thus, for instance, in Austria you might have mathematics.

as one principal subject and Darstellende Geometrie for the second, with Physics as accessory subject. This would not be possible in Prussia. In Prussia, if Mathematics is the principal subject, two accessory subjects must be chosen from among the following: Physics, Astronomy, Chemistry, Philosophy, Classics, Natural Science, Geodesy (?), Engineering (?).

This is what I gather on the subject of Austria from V. Sterneck, he is not however quite clear on the subject, as he passes quickly over the Doctor examination and discusses more at length the Staatsexam (to be considered immediately), apparently on the grounds that it is rare for a candidate for the Doctor's degree in Austria to take Mathematics as a principal subject. In Vienna in the ten years from 1900-1909 there were only 68 such, and in the whole of the rest of Austria only 19. V. Sterneck remarks that this is deplorable and points to the fact that people only take mathematics as a means of earning their livelihoods; in fact, as soon as the mathematical students have passed their Staatsexamen, they are able to take a post at a school, and any further study may be regarded as a luxury.

It is particularly to be remarked that this examination is not a competitive examination. The candidate may be unsuccessful (durchfallen) or he may pass (1) *summa cum laude,* (2) *magna cum laude,* (3) *cum laude* or (4) *rite,* but the signification of these terms is exceedingly relative, and little weight is laid on them as such. Thus, for instance at one University a “summa” may be frequently given, while at another it may be hardly ever given, and yet a “magna” there may be more difficult to attain than a “summa” at the former place. On the other hand a great deal depends on the individual professor. Thus for instance in the years at the end of the preceding century Professor Klein hardly ever gave a “summa”. He said, in my opinion quite rightly, that he must keep something in reserve for a possible Riemann. He gave a “summa” to a young man of great promise called Ritter, who died unfortunately before producing any great work, but whom his fellow students regarded as the best of them, and Professor Klein would not give a “summa” to Miss Chisholm in 1895, although the professors of Physics and Astronomy (as I know from personal communications of the three professors concerned), wished to do so, on the grounds that, the dissertation was not of a sufficiently important character to justify so high an honour. On the other hand in the subsequent years Professor Hilbert, who took Klein’s place was more free with his “summa’s,” an awarded for instance a first class to Max Dehn, which caused some dissatisfaction.

The so-called Staatsexamen or Lehramtsprüfung is the necessary qualification for the exercise of the calling of a teacher in Germany and Austria. It is not a University examination and is held by a commission specially appointed by the state for that purpose, the members of which are for the most part University professors but do not exercise their office in that capacity, but as state officials, appointed for a definite task. It is usual in Germany to take the Staatsexamen first, and then, if the candidate wishes it, to go on to the Doctor examen ; but the work for the two examinations goes on simultaneously, and the original work used for the Doctor dissertation is commonly used first for the Staatsexamen. The Staatsexamen consists of three parts: (1) the dissertations, which are, I believe, two or three in number; they are not in general of such a high order as the Doctor dissertation, and commonly are of a historical or critical nature; they are not obliged to be so long as the Doctor dissertation, and they are in general not published, * nor do copies of them have to be sent to any but the examiners. (2) The so-called Klausurarbeit, or time-paper; this does not occur in the Docotorexam at all; the candidate has certain questions given him to answer, or rather to work out, in certain fixed periods, *viz.*, two periods each of four hours with a pause for dinner, his principal subject or subjects for each such for hours for each accessory subject. (3) A viva-voce examination in the subjects which the candidate wishes to teach.

The number of principal and accessory subjects is not, as in the Doctor examination, fixed; for instance V. Sterneck gives the following possibilities in Austria :

(d) Mathematics and Physics as principal subjects.

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* In Austria it is not obligatory to publish the Doctor dessertation The best one however, gen'lrally inserted in the Manatshefte (Verna) In Germany the publio st.ion is obligatory.

(e) Philosophy conjointly with Greek as principal subject and Latin as accessory subject; *or* Philosophy and Mathematics as principal subjects and Physics as accessory.

(g) Natural history (Science) as principal subject, Mathematics and Physics as accessory subjects.

(h) Mathematics as principal subject together with Darstellende Geometrie as another principal subject, or with geometrical drawing and physics as two accessory subjects.

When mathematics is taken as a principal subject the candidate is required to show knowledge of arithmetic in general, synthetic and analytical geometry, Differential and Integral Calculus and their applications to geometry, the elements of the theory of variations, and the elements of the modern theory of functions.

When mathematics is taken as an accessory subject, knowledge is demanded of elementary mathematics, particularly those parts taught in the gymnasium and the Realschule, and the candidate is expected to show that he is able to manipulate the methods of calculation and of construction required in those subjects with absolute certainty and confidence.

Before obtaining permission to take the examination the candidate has to show that he has been four years an immatriculated student at the University; only under certain circumstances is this rule relaxed, and then only so far, that the candidate is permitted to teach at the inferior schools only.

The following list of dissertations taken at the

Staatsexamen, (Hausarberit), in Austria is interesting:

(a) For the principal subject J.

The Theory of Fourier Series.

The isoperimetric problem.

Systematic Development of the proofs of the fundamental theorem of Algebra.

Abel's Equations.

Theta functions and their application in the theory of surfaces of the 4th degree.

Historic development of the progress which has been made in the theory of algebraic equations on broad lines.

The solution of algebraic equations by means of transcendental functions.

The partial linear differential equations of the second order (with special consideration of those types which occur in mathematical physics).

The formation of invariants of ternary forms.

The triply orthogonal systems of surfaces.

The development of elliptic functions from the differential equation.

Transcendental numbers, particularly e and h.

The Jacobian, and its most important applica -tions.

The substitutions of five elements.

Theory of the lines of curvature of a general ellipsoid, with a representation of these lines.

The integral functions with respect to a prime modulus.

The methods of obtaining very large prime numbers.

The series for tan x and see x and the most important properties of their coefficients.

The importance of the number in the calculus of probability and the theory of errors.

The most important definite integrals which can be reduced to Euler's Functions.

The focal properties of surfaces of the second degree.

The most important theorems and formulae in algebra which can be applied to congruences with respect to a prime modulus.

The series of Fourier with special reference to the works of Fejer's and Lebesgue.

The representation of continuous functions by series of polynomials.

The connection between Theta and Sigma-function.

The algebraic discussion of the 27 straight lines on a cubic surface.

Functions with gaps.

The application of elliptic functions to geometry.

The progress which algebra has made, thanks to. Lagrange, Gauss and Abel.

Modern investigations into integral functions.

The singular points of algebraic curves.

The comparison of the concepts of integration due to Riemann and Lebesgue.

Existence of a solution of an ordinary differential. equation, and graphical treatment of the same.

Reye's complex.

Confocal surfaces of the second degree.

The cubic surface and its representation on plane.

The differential equation of Lamé.

The equation of the fifth degree.

The twisted quartic of the first species and elliptic functions.

The rational twisted quantic.

The problem of the principal axes of plane sections of a surface of the second degree.

A study of the representation given by $w = \int dz / [(z-a)\,(z-b)\,(z-c)\,]^{1/3}$.

The line of the real integral curves of an ordinary differential equation of the first order (where y' is a rational function of x and y) in the neighbourhood of its singular points, and in like manner those of a system of two simultaneous equations of the first order is to be represented according to the fundamental work of Poincaré.

Tavary's Theorem in plane Kinematics and the constructions which depend on it are to be developed and illustrated with examples.

The elements of the theory of surface is to be developed on the assumption that the polar coordinates of a point are given as functions of two parameters.

In the theory of plane curves, the most important concepts and theorems are to be developed, which involve differential coefficients or are of order higher than the second.

The general theory of surfaces is to be applied to the anchor ring.

Theory of linear integral equations and small oscillations.

Analogy between linear ordinary differential equations and algebraic ones.

The chief properties of the one-valued doubly periodic functions.

Surfaces of constant curvature.

On infinitesimal transformations and their application to differential equations.

The theorems of Mittag Leffier.

The behaviour of a power series in its circle of convergence and on the circumference.

Discussion of Riccati's differential equation.

The different modes of representing elliptic functions.

The polar properties of curves and surfaces of the 2nd degree.

Number systems with more than one complex unit.

The metrical properties of surfaces of the second degree.

The Reciprocity theorem for quadratic remainders, particularly its connexion with the problem of the division of the circle.

The numerical solution of algebraic equations.

Infinite products and their most important applications.

The transformation of the variables in multiple integrals.

The representation of integers by means of binary quadratic forms.

The most important algebraic and arithmetic applications of continued fractions.

(b) for the accessory subject.

The solution of equations of the 3rd and 4th degrees.

The properties of the Nine-point-circle.

The filling up of space by spheres in a heap.

The theorems of Pascal and Brianchon and their proof in the case of a circle.

The series of powers of the first integers.

The general term of Lamé's series and the theorem about the greatest common factor of two terms.

The moments of inertia of the regular tetrahedron, octahedron and hexahedron, When one of the edges is the axis of rotation.

The different number systems, arithmetic operations in these, the transformation of numbers in the various systems, the use of divisibility, especially for the bases 4, 10 and 12.

The tactic problem of Hypollonions, treated by the methods of elementary

A die of 20 mm (side) is known to be such that its centre of gravity is not the centre, but lies 2 mm towards one of the faces. What is the probability for each face of coming upwards when the die is cast.

In a regular ten-sided figure each vertex is joined to the third next. Find the area of the 20 triangles so formed, and of the 10 quadrilaterals and the ten-sided figure.

Construct an ellipse by means of its points, given a pair of conjugate diameters and the analytical theory. Do the same for the hyperbola.

The Systametic Development of The Ideas of Power

The contents of Girad's "Invention nouvelle en l'algebre" (1629), is to be expressed in the language and symbolism of modern mathematics from page C 2 to page G.

The theory of periodic decimal fractions, including that simpler theorems as to the length of the period, is to be developed and illustrated by examples. How did the ancient Greeks carry out their numerical calculations ?

The theory of finite continued fractions, including their application to diophantine equation.

The formulas of arithmetic with abbreviated decimal fractions is to be discussed in reference to the accuracy required.

The elementary theory of inversion.

The theory of symmetry given by Mobius, considering the mistake in it pointed out by Schoenflies.

Paper constructions by folding according to Row.

To express the surface of the 15 Archimedian bodies, by means of the radius of the sphere circumscribed to them.

The construction of triangles, quadrilaterals, pentagons, and hexagons inscribed in a circle from the point of view of geometrical drawing.

Arithmetic series of higher order.

The representation of real numbers by systematic fractions and continued fractions.

Weierstrass's Theory of irrational numbers.

The theorems of Fermot Willson.

In the time of paper and the viva voce examinations, weight is laid on clearness and decision in dealing with the fundamental parts of the prescribed subjects, but not so much on the retention of details. In Algebra the theory of symmetric functions is required, as well as the algebraic solution of equations up to and including the fourth degree, and better candidates are expected to have some knowledge of the theory of groups and of the impossibility of solving the general equation of degree higher than the fourth. Questions are not set in these parts of the examination on the theory of variations.

In the time of paper examination it is to a certain extent required that the candidate should show that he is able to apply his theoretical knowledge to practical examples. Again in the examination special stress will be laid in the subjects which have been expounded by the professors during the period of the candidates' study at the University. This last remark applies equally to the Staatsexamen and the Doctorprüfung, and in fact, to all examinations at German universities and schools.

The following remarks of V. Sterneck are of interest :—

It may be asked, what changes should be introduced into the program of the Staatsexamen in respect of the matter on which the candidate is examined. It has been suggested that THE

FOUNDATIONS OF ARITHMETIC AND OF GEOMETRY should be demanded explicitly, as well as THE ELEMENTS OF ALGEBRA and THE ELEMENTS OF THE THEORY OF DIFFERENTIAL EQUATIONS. At present the latter is introduced under the heading of integral calculus.

The experience which we have had of candidates taking mathematics as principal subject, tend to show that the foundations of analysis, the successive introduction of the notion of number, and the extension of the laws of arithmetic to the new numbers successively introduced present more serious difficulties to the candidates than the succeeding portions of the subject. In spite therefore of the importance which these have for the teacher, many examiners avoid such questions altogether. The minds of the candidates are not sufficiently ripe for insight into these fundamental questions. This is still more the case when we come to the foundations of geometry from the modern standpoint.

If we ask whether this want of knowledge, which is generally recognised, with regard to the foundations, particularly those of analysis, will not prove a hindrance to the candidate in his professional career as a schoolmaster, It will be found that this cannot be denied altogether, while on the other hand it is easy to overrate the importance of these fundamental questions in teaching. In fact, as we have seen by these examinations, the study of these questions even at the university presents difficulties of no mean order to the student, so that there can hardly be a question of introducing them seriously into the school curriculum. Rather let us take advantage of intuitive methods such as suggest themselves at once for positive and negative numbers, and for irrational numbers by means of stretches in geometry. Gauss and many other mathematicians were far from possessing the modern theory of irrationals, and yet they constantly used irrational numbers, and that quite rigidly. In teaching in the schools one must necessarily use the intuitive method, the abstract theory of irrationals being quite unsuitable.

It is quite different with respect to complex numbers. If the candidate does not properly understand these, it is to be feared that he will when teaching, give hazy and very confused explanations to his pupils. The school textbooks increase the evil by introducing $\sqrt{-1}$ simply as a new number, immediately after proving that, all square numbers are positive. The principle, that you may regard an impossible problem as in any sort solved by the introduction of the symbol $\sqrt{-1}$ or i, applied blindly, leads to the most nonsensical results, and in point of fact people who have gone through that schools have usually the impression that imaginary and complex quantities are pretty ridiculous and superfluous things.

It seems therefore, that special weight ought to be laid on a complete comprehension of the introduction of the complex variable in the Staatsexamen, that is if the theory of the complex variable is to be retained in the school curriculum, which is the case in the programme for 1909. Here the intuitive method fails, (unless, which would perhaps be quite good, the points of the plane be regarded as the representatives of the complex variable). We must therefore make up our minds to proceed by abstract reasoning, introducing in some way, perhaps a quite special way, pairs of numbers, and explaining clearly the operations underlying calculations with these pairs of numbers. Simply to write down the number-type a+bi, and to make the convention $i^2 = -1$ is nothing at all. It is necessary to make sure that the future teacher gets quite clear ideas about these things.

It is easy to convince oneself that the usual presentation of the subject in school books is unsatisfactory and only arouses suspicion, by considering what happens in the examination. It constantly happens that a candidate, who have got up their elementary work otherwise satisfactorily out of such books, are found to be quite at sea as soon as they are asked about the complex variable. Recently the author of this report (V. Sterneck) was examining a candidate taking mathematics for an accessory subject, and who seemed thoroughly well prepared, and explained quite nicely about the complex variable, just as it stands in the book A. Being asked whether he had any clear notion of these things, or whether he looked upon them as an arbitrary, more or less nonsensical kind of game, he confessed to the latter opinion. This was certainly a proof of a higher order of intelligence, implying that the candidate at least felt the gaps in his knowledge and the want of grasp of preliminary ideas. When I pointed out that theoretical Physics and other branches of science, have certainly to do with real life, frequently use the complex variable in their calculations and that the results agree with reality, he declared that he had no idea how that came to pass. From such experiences it is to be concluded that a careful training in these things is necessary for candidates taking mathematics for principal subject, as they will themselves have to refer to the complex variable in their school teaching.

It is different with the foundations of Geometry. Here the difficulties may be said to have been introduced subsequently. The ideas of a point, a straight line and a plane are connected with quite clear representations in the mind of even, the most uneducated, and the axioms of Geometry, as they have been formulated from old time,. are really simple and even self-evident truths. If it is perhaps desirable to speak in the schools of the omission of the parallel axiom and the possibility of Non-Euclidean Geometry, yet the subject of the foundations of geometry, so much is certain, has no place in school teaching. For this reason this difficult subject need not be afforded such attention in the Staatsexamen and its introduction into the requirements would certainly occasion very undesirable. results.

In the examination for the accessory subject, that is in the case of candidates for teaching mathematics for posts in the lower classes of the schools, the demands of the examiner do not usually go beyond what was already required in the Maturitätsprüfung. Some examiners however demand spherical Trigonometry and perhaps the solution of the cubic equation, or lay stress on the development of the Geometrical Faculty in three dimensions. The preparation of these candidates is in general by no means good, since the science students in particular have little time for the study of Mathematics, their time being already overful with naturl-science, and at most Universities there is a want of elementary lectures, so that the preparation has to be carried on by means of text-books. For these reasons, the results of the examinations are often by no means brilliant, even moreover really capable science students have sometimes not alert whatever for Mathematics, and yet, for extraneous reasons, few are obliged to take Mathematics for an accessory subject. It would however be a mistake to say that all the experiences in this respect are bad. There are some very good exceptions among the candidates.

The inefficient preparation of these candidates has convinced many that the whole institution of the Mathematical accessory subject is a mistake. Thus Professor V. Dantscher in Graz and Professor Gmeiner in Innsbruck are of opinion that every candidate for permission to teach Mathematics should have to take that subject as a principal subject. This would certainly raise the level of Mathematical teaching in the schools, but, as Hofrat Mertens and others point out, it seems to be impracticable, as the requisite number of Mathematical teachers would scarcely come forward. It seems therefore as if the step to be taken is to provide adequate elementary lectures for these candidates. It is evident that the study of text-books cannot be sufficient even to freshen up the knowledge which the candidate possessed at the time of his Maturitätsprüfung. It is therefore to be hoped that such courses will be introduced, as is already the case at the University of Graz, so that every student taking Mathematics as an accessory subject may have the opportunity of obtaining an introduction into the subject suitable for his needs and his abilities. This would certainly have a beneficial effect on the examination, since it is clear that the matter can be handled at the University more fully than at school.

Libraries etc. in Austria

It is only at Vienna that there is a really satisfactory Seminarbibliothek. These are about 1528 volumes (in 1911), counting 60 current periodicals, exchanged for the Monatshefte für Mathematik and Physik. The grant to the seminar for the increase of the library is 1480K. At the German University in Prague there is only 400K a year for the Mathematical Seminarbibliothek, and this includes all that is available for Mathematical Physics; this is of course quite inadequate. In Graz it was the same till quite lately, when a representation of the committee of the Seminar induced the Government to raise its grant to 600K, so that 400K is available for the Mathematical department alone, without Physics. The Mathematical reading room in Graz contained till lately only 180 books, so that it is hardly worthy of the name. In Innsbruck the reading room contains 575 books. In Czernowitz there is no regular grant, and the reading room only contains a large bookshelf. In Krakau and Lemberg the more important text-books are to be found in the reading room.

The library at Vienna contains about 1,05,000 Mathematical books. The yearly expenditure in new books is about 1600K, and about 220 volumes are added yearly. The grant to the library in this respect is therefore comparable to that for the Mathematical reading room. Besides this, the Polytechnikum has an excellent Mathematical library, and in particular a splendid collection of periodicals. In the reading room of the Polytechnikum there are 47 periodicals. In Prague also the library of the Polytechnikum is rich in books and periodicals, while the University Library is said to be deficient in Mathematical literature. In Graz there were in 1911 about 300 Mathematical works in the library, this includes a number of dissertations, the number of current periodicals is 8; the yearly expenditure about 500K, and the number of volumes added each year about 110. Here also the library of the Polytechnikum is good; the number of current Mathematical periodicals is 21. In Innsbruck Mathematical literature is well represented, owing to an extraordinary grant of 4000K, which Professor

Wirtinger obtained in 1898 and 1899. Czernowitz and Krakau are very badly off in this respect; but at Lemberg the older literature was well represented, and the library of the Polytechnikum was relatively rich in Mathematical books and periodicals (how much of this is now left)?

Translation of the Appeal for more Mathematical Chairs, made by the Austrian Universities to the Government (1907).

The number of University teachers in Mathematics, and consequently the whole organisation of the Mathematical teaching in the Universities is so far from corresponding even to the most modest requirements of modern thought, that the proposal of the University of Cracau that all the Philosophical faculties of the Austrian Universities should join to draw up a report on this subject and lay it before the Government, met with unanimus approval.

In the last half century the demands made on the Mathematical department of a University have completely changed and have become much more manifold.

If the Mathematical teaching at the University is to correspond to the modern exact requirements, it cannot be merely the continuation of the teaching at the schools; on the contrary the elementary parts of Mathematics must be taken and systematically developed, perhaps shortly, but in a rigidly scientific manner. Thus there ought to be a certain number of University teachers engaged in the teaching of elementary Mathematics.

But further, new branches of Mathematics have arisen. The scientific importance of these new branches renders it imperative that they should be introduced into the regular curriculum. Who could contemplate leaving out of the programme of the Mathematical department of the University, such subjects as the modern Theory of Functions or synthetic Geometry ? But the teaching of such subjects necessitates an increase in the number of hours of Mathematical teaching. As however the deductive character of Mathematics leaves little liberty in the order in which the different courses of lectures follow one another, it is desirable that the elementary classes should be held every year, all the more, because otherwise the students would not be able to follow the instruction in Physics. This is only possible if the number of teachers is increased.

But also for other reasons the study of Mathematics, regarded as a principal subject, demands a greater number of hours and of teachers. Mathematics, like Natural Science formerly and Historical Science at the present day, has branched out in so many directions, that it falls into a number of different subjects, different in matter and treatment. At the lowest computation Pure Mathematics must be divided up into the following domains, each exceedingly far-reaching:

(1) Theory of Numbers and Higher Arithmetic.

(2) Higher Analysis, embracing Differential and Integral Calculus, Differential Equations, Theory of Variations, Theory of Functions, etc.

(3) Analytic and Synthetic Geometry, including the theory of Groups.

Besides these branches of Pure Mathematics there are those portions of Applied Mathematics to be considered which can only be properly treated by professional mathematicians, owing to their close connection with Pure Mathematics. Not counting Theoretical Astronomy and Mathematical Physics, we may, in particular, note the following :

(1) Theory of Probability with its various applications.

(2) Analytical Mechanics, and

(3) Darstellende Geometrie and the various kinds of Geometrical drawing and Graphical methods (Graphical Statics, Graphical methods of integration, etc.) as of great value in training the Geometrical intelligence.

When we consider that, with the exception of a very few men of exceptional ability, a single man cannot be expected to be a master of more than one of these domains, and when we reflect that only one who has himself really. produced some original work in the subject in question, is capable of giving the requisite stimulus to his hearers, and of introducing them to the proper methods of research, we shall not hesitate to say that two professors cannot possibly suffice to bear so great a burden.

If we regard Mathematics as a help in other subjects, it must be recognised that the importance of Mathematics for the other sciences has enormously increased in the last half century. Mathematics with Physics, has penetrated deeply into all branches of Nature study, and is penetrating deeper every day. It ought really to be considered essential for every body to understand something of Thermodynamics, and theoretical chemistry is really only comprehensible to some one who is familiar with Thermodynamics.

But there can be no question of understanding Thermodynamics unless the student has some knowledge of differential and integral calculus. Thus it is essential that the students of natural science should have the possibility of gaining some insight into higher Mathematics. But none of the courses which are arranged for those who are taking Mathematics as principal subject are suitable for such a purpose. The treatment of Mathematics for students of other subjects must be a quite different one, owing to the small number of hours available. Much more weight must be laid on intuition; and many notions and theorems must be taken, not in their full generality, but with certain convenient restrictions, so as to make them easier. Such a treatment has already found its place in print; indeed a number of Mathematicians of the first rank have produced text-books of Mathematics for non-Mathematicians in German, French, Italian and English, (Lorenz, Appell, Burkhardt, Nernst and Schoenflies, etc.). Summing up, it is necessary to have an organisation independent of that for students taking Mathematics as principal subject.

In order to cope with all these requirements at least in part, it is essential to have at least three professors, and this number will only be sufficient if two of the Chairs have also an assistant. In fact the elementary exercises corresponding to the courses must be looked after by the assistant under the control of the Professor. For, if the exercises are to be really profitable, they must be suited to the individual needs of the student, as is the case in all practical subjects and in darstellende Geometrie at the Polytechnikum. That is to say, it is not sufficient to work through on the blackboard one or two examples in the presence of one or more students, but each student must have a selection given to him to choose from and they must work these out for themselves under the personal supervision of the Professor and his assistant. Independence, even in a modest degree, is an essential for the future teacher, if he is to possess the requisite self-confidence and energy in his teaching.

As it is evident that the whole body of education in a country depends principally on the teaching at the Universities, and that therefore a good organisation of the University teaching is of fundamental importance for education in general, the philosophical faculties of the Austrian Universities are of opinion that it is imperatively necessary to take steps to satisfy the wants of the Universities here set forth.

There ought then to be in the philosophical faculties of each of the Austrian Universities at least three mathematical Chairs for teaching the following subjects :—

(1) Theory of numbers and higher Algebra;
(2) Mathematical Analysis;

(3) Geometry.

At least two of these Chairs should be "ordentliche Lehrkanzel", and each be provided with an assistant.

As however there are also Universities where scientific studies are very largely pursued, it is desirable that, besides the Chairs and assistants above mentioned, the philosophical Faculties of the greater Universities should be provided with the following Chairs :—

(1) A second Chair for higher analysis, (ordentlich) ;

(2) A Chair of Applied Mathematics (ordentlich) with an assistant; this is not provided at any Austrian University.

The modesty of these demands is best seen by comparison with the foreign Universities, which may be done by means of the following table:-

Mean number of Professors for Pure and Applied Mathematics at a single University

In		Ordentliche	Ausserordentliche	Total Number of
Austria[*]	...	1.7	0.5	2.2
Germany	...	2.1	1.3	3.4
Italy	.	3.9	0.5	4.4
France	...	2.7	1.0	3.7
Russia	...	3.7	0.4	4.1

In making this table, the honorary Professors who are sometimes to be found at German Universities, and the "Chargés de Cours" and "Maitres de Conférences" at the French Universities are counted among the "ausserordentliche" Professors.

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[*] Right. for 1910.

It is at once evident from this table without any explanation, that the conditions which reign at present at German, Italian, French and Russian Universities are considerable more favourable even than those which would reign at the Austrian, if the Government should see its way to accept the present proposals.

Under these circumstance the philosophical Faculties of the Austrian Universities hold it to be their duty to call the attention of the Government to the fact that the number of mathematical posts no longer corresponds to the increase in scentific and didatic demands made upon the Universities, and is considerably in arrears compared to the other states mentioned above. If therefore, in recognition of this state of affairs, the several Universities should present proposals for the completion of their mathematical staff, in accordance with their individual needs, the philosophical Faculties of all the Austrian Universities beg the Government, to whom is intrusted the even-handed attention to the interests of the several branches of Science, to afford its support to the realisation of these demands.

To understand the success of the German system, and in particular, its effect on the output of original work in Mathematics it is not enough to study programmes, or even to be intimately acquainted with the intimate personal relations which exist between the teacher and the taught,—relations which, I imagine, correspond more to that between the Indian seer and his disciple, though always at a respectful distance from the Indian example, than any similar ones elsewhere in Europe. While in England the young student has been awarded at the age of two or three and twenty, or little more, a brilliant or sufficiently important position, which he is destined to occupy for the rest of his life, and while in France success, though it comes later, and does not reach the corresponding degree till after the attainment of the Doctorate, a real distinction in France, rarely gained before a man is well in the thirties at least, has usually the first effect of interfering with the productivity, an interference which is more or less intermittent till one of the really great positions in Paris has been attained, in Germany there is a steady and continuous incentive in favour of research from the moment the student enters his first University as a freshman. Even in his young student days, his studies are rarely confined to one single University. It is indeed not the University which attracts him, but the eminence of the Professors who teach there. Thus he will pass from one University to another to gain inspiration from men whose names resound all over the civilized world, wherever research is pursued or learning respected. He will be inevitably led to desire to treat the path which they have trod, and from each and all of them he will gain something to help him on his way. If at first his aspirations are so modest that he aims only at becoming a teacher in a secondary school, the regulations of the examination which he requires to pass are such that he must at least write a thesis on some subject which requires elucidation, or whose known results need to be collected. Gaps in the existing knowledge are thus laid bare to him, and a way of approach to the innermost mysterier of human knowledge is thereby often laid bare to him. Should he be encouraged in consequence to pursue his studies further, with the view of obtaining the Doctor's degree, the standard for this degree is not so high as that in France, only a privileged few can hope to attain unto it, and that his attempts to secure the coveted dignity, one which will confer on him a certain dignity all over Germany and German-speaking countries, he is sure of having the sympathy as well as the knowledge and experience of a competent professor at his disposal, whose help, indeed he is in the habit of acknowledging publicly afterwards, while the extent of that help is sometimes such as to leave a doubt as to whether it has not gone beyond what is reasonable and right. If his dissertation has been a sufficiently encouraging one he may have been awarded with a "Summa cum laude." But a smaller degree, a "magna cum laude," a "cum laude" or even a "rite" will not necessarily bar his way. A comparative failure at this point need only prove a temporary stumbling block; it will never be counted against him in years to come, should he continue on the professorial path. On the other hand should he go no further, he has gained a respect for things of the intellect, such that he will remain all his life interested in scientific matters. The elite of the German Universities will rely on him and such as him, form an audience fit and not too few. Moreover they will hold up the beacon themselves, in so far as in them lies, even in the remotest parts of the Empire, to which their life's work may call them, whether as medical men, clergymen, lawyers or school-masters, or even as booksellers and chemists. Among other things the existence of this comparatively large class has the excellent effect of rendering charlatanism less practical and less profitable than it is still found to be in England and in the United States.

The gaining of the Doctor's degree, meaning as it does less than in France, does not at once open the doors to professorial success. To be allowed even to give a voluntary course of lectures at a German University, more than this is required. A further, and usually a more important piece of research must be undertaken and specially approved by the Faculty of the University at which the Doctor wishes to become a Privat-Dozent. There is usually a limit to the number of Privat-Dozents at in a particular subject at a particular University, and there is a special proviso that only such Doctors should be allowed to become Privat-Dozenten who, it is judged from the nature of this second dissertation, the so-called Habilitationsschrift, among other grounds is not likely to remain merely a Privat-Dozent for more than a comparatively short period. For the rest as the Privat-Dozent is not a servant of the State, he receives no salary, and the position he has obtained so far offers no guarantee of itself for further advance a man is not likely to continue unless his interest in his research work and his capacity of dealing with it are such as to afford him a reasonable hope of further advancement in the not far distant future. In certain cases he may receive a certain small stipend from the University, on the understanding that he gives a certain course or courses—this is so in some of the countries which have imitated Germany in their University system. He also receives a certain portion of the fees of students attending his lectures. But it is only in rare cases, and then only as a personal favour, that he gives private lessons. To do so in fact more than is absolutely necessary might shipwreck his whole career. On the other hand the number of private pupils he might so obtain would not be such, that, even if he cared to affront public academic opinion by attempting it, he could be sure of even a moderate livelihood.

As Privat-Dozent then, although already of an age at which his English rival has in the past of ten already entered on the duties which he will pursue to his death with an assured income, and with little or nothing to show in the way of work done, the German has still his way to make. He is only on the very outermost fringe of the German academic world, and whether he is able to remain a University teacher at all depends wholly on the success which attends his efforts in the next few years to add to the sum total of what is so far known in the subject to which he has devoted himself. No success, however brilliant, in helping more or less indifferent pupils to pass their examinations will be of the slightest use. He will be judged almost certainly on his record of work done. Even social connections, whether at court or in University circles will not serve, unless the groundwork of research which counts, is there, for his friends to point to and make the most of they can. If his work is really brilliant, or even possesses merely a certain importance or a certain interest, nothing can stop him ; not even the reputation of having few friends. In no case will a man who has done little or nothing be preferred before him; or, if this should, very exceptionally occur, it cannot continue to do so; in the long run the future of such a man is assured at least up to the next stage, and probably beyond.

This next stage is that of becoming an extraordinary Professor (ausserordentliche Professor). By the time the German has attained this rank, his English rival has even now in many cases and it was worse in the past, already begun to tire of the endless round of elementary tuition, elementary examining, haunting of committee meetings, Faculty Meetings, Senate Meetings and the like, of which his life is made up, and has almost always more or less definitely renounced the ambition, if he ever had one, of producing original work. The most that he is likely to attain unto, is the writing of a text-book, embodying, it may be, what he acquired in better days, or it may merely be a printed account of the matter which he has found his pupils are able and willing to assimilate.

The German extraordinary professor becomes so usually by a call to another University. Such a thing as a candidature initiated by the man himself, is, I believe, unknown in Germany. One of the extraordinary professors of another German University, or even a German-Swiss or Austrian University, has received promotion, probably by a call to another University. The Faculty to which he belongs meets and if the University is in Prussia, two names are selected from among the Privat-Dozenten of German Universities, known for the work they have done. These are sent up to the Minister of Education (Kultusminister) who usually selects that one of the two men whom the Faculty prefers. Either before or after the sending up of these names, the candidate preferred by the Faculty is formally or informally, as the case may be, invited to accept the post. This invitation or "call" is not always accepted; a man of great ambition and conscious of great ability, will sometimes prefer to remain as Privat-Dozent in a great University and in an inspiring milieu, rather than to go as extraordinary professor to one of less reputation where he will be isolated, and may be relatively badly off in the matters of libraries and other instruments of research. More often the invitation will be accepted, as it is unusual, and if not wholly unprecedented, regarded as wholly undesirable, for a Privat-Dozent to become extraordinary professor in his own University, without having been in the meantime extraordinary professor, or at. least Privat-Dozent at another University. Occasionally a Privat-Dozent as has just been stated, refuses however, a call, and is only induced to leave his University by a call from a

Great University as extraordinary professor, or, very exceptionally, full professor.

As extraordinary professor the German man of Science is at last a Government Servant, and enjoys the benefits accruing to this position. Though this was not always the case, his wife in the event of his death has a small pension, and though his emoluments are small, his position is at least an assured one. As extraordinary professor, however, he has not usually or always a place on the Faculty, and is in any case more or less under the authority of the ordinary professors, if not always theoretically, he is practically under the necessity of lecturing on the subjects selected by them, and of disposing of a considerable part of his time as they may direct. The better organised the University, the more is this likely to be the case.

The only way in which an extraordinary Professor comes a full professor is either by the retirement, by death or otherwise, of an ordinary professor at his own or some other German University, or by the creation of a new post of this kind a contingency which. not infrequently occurs, owing to the constant expansion of the German nation, and the increasing number of students at the University. Usually he will not become ordinary professor at the University where he is extraordinary professor, unless he at least has received a call elsewhere. It is not uncommon, if he is anxious to remain where he is, for the extraordinary professor to go to the Faculty with the invitation in his hand, in order that he may give them the opportunity to arrange for his retention, if this is both desirable and possible. If the University is anxious to retain him, it will usually be necessary to create a new Ordinariat, and to get the necessary permission from the Government. In exceptional cases the stipend of the extraordinary professor may be considerably raised so as to compensate him for the pecuniary loss which his refusal to go elsewhere might otherwise cause him. Usually nothing less than the Ordinariat will induce him to remain.

As ordinary professor in fact the German man of Science first begins to enjoy the full advantage of the position with which his career has provided him. He is now for the first time to all intents and purposes independent. His means are less narrow, he is treated with the utmost respect wherever he may chance to find himself, and, what to do him justice he appreciates even more, he is entirely free in the matter of his own researches. Moreover not only is he, like the extraordinary professor, but little burdened as regards the number of the lectures he has to give per week, his choice of a subject to lecture on is to all intents and purposes entirely in his choice, within the limits, of course, of the subject which he professes. There is no examination whose range he has to consider, for which the men attending his lectures may be preparing, no reproach can ever be urged against him because men attending his classes fail to answer questions set by others. And so far is he expected to attempt to teach whose abilities are unequal to the task of studying alone, that it would be regarded as a disgrace to any professor if he had unripe students attending his lectures, equally whether he had made an effort to enlist them, or not. His reputation, his position, his future depend, even more than that of the extraordinary professor, on the success of his efforts in extending the horizon of knowledge, whether directly by his own research, or directly by the efforts of his students, to whom he stands as guide, philosopher and friend. The very last thing that is expected of him is that he should encourage to continue in the career of letters or of Science those who are unfitted by nature for them.

Should at the age at which a German scholar has become an ordinary Professor, an English man of Science be still unattached as professor to an English University, and have the audacity to present himself as a candidate for a Chair, and to go through the necessary proceedings, including the soliciting of numberless testimonials and private letters of recommendation, with, in Scotland, the additional burden of visiting the Members of the Court of Electors individually in their own homes, he will probably be rejected on the ground that there is no evidence that he is a good teacher, or able to control the undisciplined classes of men of small ability, who, though they do not much wish to learn, desire at least to gain the degree, the absence of which would reflect discredit on themselves and their friends, as well as interfere with the career they have marked out for themselves. A suggestion that his not having held such a post hitherto is due-to his not being in favour with the professors, in the other universities, many of whom are likely to be quite incapable of judging of his work, whatever the subject he professes, is likely to be equally fatal, and the man chosen, in the case when a younger man is not desired, may well be a man grown old in the harness of elementary teaching to large and unruly classes. A different choice the electors may urge would deplete the University or too many of its numbers, or at least entirely change the character of the particular department.

It is not surprising that the advantage is all in favour of the German professor in the matter of research. But this is not all. By becoming ordinary professor he may have already become head of the school of mathematics at his University, or his senior colleague or colleagues may, so far as the work for the Doctorate is concerned, hand over the control of it to him. In this way he not only, as during the time when he was extraordinary professor may have been already the case, has the supervising of the dissertations of particular students, he has now so complete a control that, with few exemptions, no dissertation will be produced in his department of which the subject has not been chosen by himself. He will naturally choose those branches of Mathematics in which he is himself interested either by having a special knowledge or as constituting fields in which he has himself worked, or finally which contemporary work elsewhere has shown to be fruitful. He is no longer an isolated soldier but is in command of a regiment, and it will be expected of him that he should make full use of the weapons thus placed at his disposal.

The professor on the other hand who either through lack of sufficient ability or indolence rests on the laurels he has already gained, will find his social position among his colleagues inevitably affected. Even as ordinary professor and even if he is perfectly content with the position that he has gained, public opinion will render it difficult, if not possible, for him to discontinue his private researches. He will probably wish, however, to possess, like his more distinguished colleagues, various tokens of official and public recognition. He will aspire to become a Privy Counsellor, to have an Order, to become member of the most famous foreign and national Royal Societies, and the like, even if his zeal for Pure Science is not such as to render him a disinterested devotee.

If he knows how to use it, and if the school over which he has been called to preside is a great one, it will be far easier for him to prosecute successful investigations than ever before, especially if he be a mathematician. The time spent on the mastering of memoirs written by others necessarily quite incommensurably large in the case of a mathematician, however disposed he may be to reduce this to a minimum. In Germany in particular public opinion demands that other peoples work should be cited. and cited correctly, while the difficulty of making oneself acquainted even with the contents of mathematical papers, to say nothing of the details of the argument, is difficult beyond all comparison to what is required in other subjects. The professor will have the advantage of being able to entrust the digging and delving operations which this involves both to the younger and more mature of his students in the full strength of youth. The information he thus gains will largely compensate and if the school is a great one more than compensate for the time lost in the routine, whether in connection with the work of the school or in the organisation of the University and Department. Thus, to give a well-known example, Klein, who by dint of an extraordinary genius for popularising, or rather rendering accessible to ordinary mathematical students the most abstruse mathematical subjects, had transformed school consisting of a handful of students into one where they are counted by hundreds, conferred on Hilbert, in handing over to him the organisation which he had cteated advantages which cannot be exaggerated. It is not too much to say, and this in no way depreciates the value of Hilbert's work, that there must be many men in Europe, as well as some in America, who would have made no less able a use of these advantages. In point of genius indeed there are a number of men of Hilbert's generation who have produced less work and posterity, even as it is, will probably place higher, and these among men whom the popular voice even outside Germany would probably not place on the same rank with Hilbert at the present moment.

When the professor has become an ordinary professor, his material position may still be improved. He may receive a call to another University, where the emoluments are greater, and will in either case, whether he accepts the call or refuses it, henceforth have a higher income. Moreover the greater his success, the greater will be his income from his portion of the fees, surrendered to him by the University. If all that he cares for be the improvement of these material conditions, he has still every reason to continue his researches.

The other reasons of a somewhat higher nature for doing so have been already referred to above, and even if, for some reason, Government neglects to honour him, his scientific work will gain for him a degree of applause from his contemporaries which will go far to compensate him for this neglect, and, however abstruse his researches, he is sure of an audience in Germany.

REPORT ON THE STUDY OF MATHEMATICS ON THE CONTINENT OF EUROPE

By

PROFESSOR W. H. YOUNG, Sc.D., F.R.S.

France :

The school education in France is terminated by the Baccalaureat, which corresponds roughly to the Examen de Maturité in Switzerland, and the Abitur or Maturitätsprüfung in Germany. This examination forms the entrance to the universitä. Nevertheless it is found that the students on their arrival at the University are not sufficiently equipped mathematically to enter at once on the course for the Licentiate, the first of the two degrees conferred by a French University. For this reason it has gradually come to pass that part of the University course, not explicitly required for the Licentiate, overlaps with part of the school course, and latterly this state of affairs has been explicitly recognised and legalised by the introduction in most French Universities, of a Certificate on General Mathematics. This preliminary certificate is demanded from candidates for posts at the better primary schools, and certain other posts, including the science posts in the secondary girls' schools (lyces de jeunes filles). It does not, however, seem to be obligatory on candidates for the licentiate and doctorate, and clever students who have profited by the instruction given in these subjects at the lycée probably dispense with it. Since 1907 also, it may be taken by candidates for the Agregation des sciences physiques instead of that for Rational Mechanics.

The situation is not however a quite satisfactory one, and therefore, more to be considered as a warning than as an example. It has been suggested (Vissiot) that a first-year at the University should be consecrated to these studies, and the certificate should be made obligatory for candidates going on to study for the licentiate; it would be easy then to give a dispensation to those students who had obtained the necessary preparation at the lycées, students who seem to be in a decided minority.

The lectures instituted at the University under this heading, though very heterogeneous in character, have proved very popular, showing that there was a want of instruction of this kind; the certificate proves too of practical value, even as at present constituted. The subjects, dealt with are:

The theory of algebraic equations,

The elements of Differential Calculus,

Analytical Geometry, Elementary Geometry, Descriptive Geometry.

Dynamics of a Particle.

It would seem therefore to correspond roughly to the first-year at Geneva. Presupposing these subjects sufficiently mastered at the lycée, or otherwise, the University teaching proper begins with Infinitesimal Calculus and Rational Mechanics. These subjects with Astronomy are the subjects required for the degree of Licentiate in Mathematics.

France. (Universities and their organisation) :

There are 16 Universities in France : Besancon, Bordeaux, Caen, Clermont, Dijon, Grenoble, Lille, Lyon, Marseille, Montpellier, Nancy, Paris, Poitiers, Rennes, Toulsouse; the Faculty of Algiers has also been created, but is not yet complete (1911). There are no Privat Docents in France; their place is taken by the Chargés de Cours, and Maires de Conférences. Nevertheless it is possible to obtain an authorisation to give lectures, so-called "cours libres." Such cours libres were given for instance at Paris in 1910 on Graphs, on Cometary Orbits and on Probability.

The French differentiate "Cours" from "Conferences." The former are open to everyone, whether belonging to the university or not the latter only to students who have passed the Baccalaureat, and are regularly entered (inscrits). Questions are sometimes asked at the "cours" but it is more especially in the conferences the students are questioned on the "cours" and do problems on the blackboard and at home under the direction of the professor. The requisite for entering the university is having passed the Baccalaureat or some equivalent examination. There is no term of years. The principle of freedom which dominates the French Universities, causes such matters to be regarded as matter for the individual student to decide for himself. The more gifted take less, the less gifted presumably more terms, and as there is no competition in the university examinations, there is no hardship for anyone. The degrees constitute a record of certain work done, and not of a certain period of residence; indeed it is not even necessary for the student to reside in France, except in so far as certain certificates have to be produced, which cannot easily if at all be obtained without having attended the appropriate classes at some time. Thus Miss Lena Spurgion, who took her Doctor's degree a few years ago at Paris, had been for some years teaching at one of the Womens' Colleges in London.

The degrees conferred by the university are the Licentiate and Doctorate. The diploma of Licentiate in Mathematics is conferred without further formality on anyone who has obtained what are called "Certificate d'études supérieures", three in number, one in Infinitesimal Calculus, one in Rational Mechanics and one in Astronomy or some one other subject. A candidate for the Licentiate may however choose himself the subjects he pleases, subject to certain restrictions according to the end he has in view, *e.g.* if he wishes to take up the profession 'of a teacher, or to go on to the Doctor's degree or for all of which the Licentiate is officially demanded. According to the subjects he chooses the Licentiate he obtains will be qualified as Licentiate in Mathematics, in Law, etc. and will serve the corresponding end.

These certificates were instituted in 1896, when the university system in France was revised, and the old licentiate, with its heterogeneities, was abolished. The certificates are obtained independently, and not necessarily in the same year. There is a separate examination for each, consisting of three parts, written, practical and oral. The written examination lasts four hours; one or more problems connected with the lectures of the academic year just elapsed are set. Sometimes also it is required that the student should write out a question treated of in the lectures, but this is always in connexion with some problem set, and corresponds, no doubt, to the book work set in Tripos papers. The enunciation of the problems set are published each year in the Nouvelles Annales de mathématiques ; it varies from university to university.

The written examination has to be passed first before the candidate is admitted to the further tests. The fortunate candidates have then to pass a practical examination lasting two hours. In the case of the certificate for Differential and Integral Calculus this consists in answering certain questions requiring no great grasp of theory but facility in calculation ; the calculations demanded are sometimes very complicated. The object is not only to test the power of the candidate for numerical work, but also to distinguish whether he is judicious in his choice of methods of approximation.

The examiners (jury) have then to decide whether or not the candidate has satisfied the requirements, taking into consideration of the written examination, before the candidate is admitted to the final examination, which is oral. The written examination is considered as carrying more weight in this respect than the practical in the case of mathematics. The oral examination consists of two parts, each of about 40 minutes. The questions asked are generally such as have been treated in the lectures, the object being to ascertain that the candidate has well covered the programme of the examination.

At the end of this final test the examiners decide whether the certificate has been earned, or not.

The degree of Licentiate is awarded to every candidate who has gained three certificates. The number of certificates organised by the various Faculties of Science in France varies from 11 to 25 according to the university. Corresponding to each such certificate, there will be a suitable professor, or professors, with appropriate courses of lectures, taking up three or four hours a week, one of which at least is a "conférence", going on during .the period of one academic year (3rd November to 30th June).

The programme is however very full, and sometimes supplementary lectures have to be given. Moreover in certain Faculties practical work of a numerical nature has been organised in connexion with the "cours"; the students solve numerical examples in class simultaneously in writing, and the exercises are corrected then and there. These are nominated "séances" in contradistinction to the "conférences", where the problems proposed require ingenuity.

In the case of Astronomy, the "cours" lasts through two academic years, one hour a week being assigned for this purpose in the universities where there is no ovservatory, while in the remaining universities, another hour or two hours is appropriated for practical work in the daytime at the observatory.

In the course on Infinitesimal Calculus, the programme of which corresponds roughly to the Cours d'analyses of Goursat, the professor, assuming that his hearers who are for the most part students in their first year have already grasped the elements of the theory, differentiation, power series (séries entieres), simple integrations, ordinary differential equations which undertake a thorough discussion of the principles, and of cases of interest which arise in the application of these principles. A chief aim of the instruction is to develope the spirit of clearness and rigour in the students, and to exercise the analytic faculty and the power of reasoning. The same idea leads to the exposition of the definition of multiple integrals and their treatment, and to the study of functions and their interdependence, questions which may be supposed to be quite new to the students.

The theory of Functions of a Complex Variable is developed according to the ideas of Cauchy. Sometimes doubly periodic functions are introduced as an application, but shortly. The existence theorem for ordinary differential equations in the neighbourhood of certain initial points is in this way led up to, and developed, following Cauchy and Weierstrass.

In integrating ordinary differential equations it is usual to confine oneself to the classical theories, including Riocati's equation; linear differential equations are specially insisted on. In partial differential equations only those of the first order are considered, and in the case of non-linear equations, it is not usual to take more than two independent variables.

In the calculus of Variations, only the principles. are considered, so that the students are able to understand that the variation of a definite integral is zero.

In Differential Geometry, the elements are considered only. Envelopes, curvature, torsion, developable surfaces, and scrolls; Meusnier's Theorem, principal sections, lines of curvature, asymptotic lines, curvilinear coordinates; in fact the programme is approximately that of the Mathematical Tripos.

"The École Normale Supérieure and the Agrétion des Sciences Mathematiques." (Rapport of M. Jules Tannery, directeur desétudes scientifiques a l'École).

The École Normale Supérieure is an institution, almost unique of its kind; it has been copied in the Scoula Normale at Pisa. It became part of the University of Paris' in 1904. The object of the school is the preparation of masters in the Iycées.

The title of agrégé des sciences Mathématiques is awarded on the results of a competitive examination held at the conclusion of the third year of study at the École Normale Supérieure to a certain number of candidates; the number being decided not only by the merits of the candidates. but also by the number of vacancies in the lycées to be filled. The curriculum of the school is dominated by exigencies of this examination. The practical importance of this examination consists in the fact that an agrégé becomes at once the recipient of a certain income and a post at a lycée with the title of professor, whereas the teachers who have not this title are only permitted to teach in the lower classes, and have a smaller stipend. This draws a number of students from the provinces to Paris, as the École Normale Supérieure is the only institution which prepares directly for the Agrégation. Though apparently it is theoretically possible to become an agrégé without having been a Normalien, but practically this possibility is never exploited. Moreover there is a tendency on the part of the authorities to advance the Normaliens at the expense of other candidates for school posts.

The agrégation is not a University Examination (examen de Faculté) but a State examination. In this sense it corresponds to the Steaatsexamen in Germany; it is however a much more severe test, and being of a competitive character, is of a totally different nature to the German examination in question.

In order to enter for the agrégation, the candidates must have obtained the Licentiate, and the Diplome d'études supérieures, the latter of which can be avoided by taking an equivalent certificate in one of the following subjects :—

Higher analysis, Higher Geometry, Celestial Mechanics, Mathematical and Experimental Physics, Higher Astronomy. These certificates can only be obtained in certain Universities, but in these they are much preferred by the students to the Diplome d'études supérieures, and in Paris, in particular, no Diplome in Mathematics has been conferred, although a number of Diplomas in other subjects have been given, and this although the number of agrégés in Mathematics far exceeds that in other branches.

Before 1907 the students could present themselves for the Agrégation five or six weeks after having obtained their Licentiate, provided they had obtained four, instead of only three of the certificates. It was felt, however, that it was desirable that they should study another year, and, if possible produce some original work, before entering for the Agrégation, which, being the opening* of their career as a teacher, might be supposed to mark the period from which on, less time for research would be at their disposal. The Diplome d'études supérieures had already been organised (1904), and was in 1907 made obligatory for the entrance to the agrégation, subject to the alternatives above mentioned.

For the diplome there are two tests :—

(1) The composition of a memoir on a subject approved of by the Faculty of Science;

(2) An oral examination on the memoir, and on certain other questions in the same branch, these questions being set three months in advance.

In Mathematics diplomas have been gained at Montpellier, Grenoble, Bordeaux, Nancy, Lyon, Marseille, Clermont, Dijon, Rennes and Toulouse. In general however the student finds the certificate a surer mode of approach to the agrégation, and students of moderate ability may perhaps gain more by following carefully a course of lectures, and submitting to the examination on them, that by spending the great amount of time required for original work, in which they are thrown back so much on their own resources. Be this as it may, the objection has been made, and from the professional point of view, in the case of medium ability, may carry weight. On the other hand, the independence of thought and the interest gained by pursuing a piece of original thinking to its bitter end are invaluable to the teacher.

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* As a matter of fact, many students take teaching work before entering for the agrégation, but it is of course of an inferior kind.

The following are some of the titles of the memoirs presented for the Diplome :—

Surfaces on which the orthogonal trajectories of a family of asymptotic lines are geodesics.

Minimal surfaces applicable to a surface of revolution.

Skew curves of the 4th order and 2nd kind.

Singularities of differential equations of the 1st order.

On Lamé's equation.

Jacobi's problem on the inversion of Abelian integrals.

Stability of equilibrium of a point in a plane.

On the rotation of the earth.

M. Tannery says that the permission to substitute a fourth certificate for the Diplome "renders illusory, or nearly so, the enactment of 1907 for the Mathematical students at Paris; previously the candidates for the Agrégation were bound to have a certificate of Mathematics other than the certificates of Differential and Integral Calculus and Rational Mechanics; they took some one of the other certificates, mentioned above. After 1907 they continued to take one of those extra certificates, which dispensed them from the requirements of the Diploma. Up to the present date (1911) none of the students of the École Normale have taken the Diploma in Mathematics. In fact the institution of the diplome d'études supéhieures, which was of real importance for the agrégations in natural and physical science, has resulted in the case of Mathematics only in the suppression of the special subjects demanded for the agrégation. Nothing has been changed in the curriculum of the two first years at the school, and it has only caused the suppression of certain lectures on science in the third year."

Having obtained the requisites for entering for the agrégation, we get to the examination itself. This consists of two parts, the first, if successfully passed constitutes the "admissibility"; the second leads to the admission of those students who obtain the best places in the examination.

The examinations for the "admissibility" consists of four compositions on problems set respectively in Elementary Mathematics, Special Mathematics, Differential and Integral Calculus, Rational Mechanics. The standard demanded is that of the licence, and the scope is practically that of the University courses on the subjects in question. Besides these written tests, the student has to execute a piece of Geometrical drawing on descriptive Geometry, and to perform a numerical calculation.

The second part of the examination consists in sample lectures, of which two have to be given in the presence of the examiners by each of the "admissible" candidates. The candidate is allowed three or four hours for preparation, with closed doors, and has to speak without books or notes. The subjects are drawn for from among a certain number chosen from parts of the programme of the Iycées whicn have been specified a year before the examination. Without being able to tell exactly what subject will fall to his share, the student therefore has had a year to revise the category of subjects from among which his lecture has to be taken, and to exercise himself in trial lectures. This system works of course far better than that previously in vogue, when the group of subjects, two of which must fall to each candidate, was itself published a year before the examination.

The programme of the École Normale corresponds exactly to the exigencies of the Agrégation. During the first year the students are subject to more guidance than subsequently, but the principle of the school is that of intellectual freedom, just as it is at the University. Two certificates are demanded from each Normalien during each of the two first years of residence; in the first year Differential and Integral Calculus and General Physics; in the second year Rational Mechanics and one of the extra subjects mentioned above.

The third year is devoted to preparation for the Agrégation itself, the examinations for admissibility requiring little more than revision, and the trial lectures being prepared, with the assistance of three or more teachers, called délégués, but really the same as maitres de conférences, a title formerly accorded to them. Throughout the three years the object of the instructors is to lead the students. to a Higher Mathematical ideal than that pursued at school. The boys on their arrival have been training strenuously for the competitive examination which gives them free quarters, rations and instructions for three years, with the possibility of a brilliant future. The teachers aim in the first year rather at broadening the culture of their pupils than at pushing them on. Their knowledge is already sufficient to render to work required for the first two certificates not unduly arduous. The lectures attended are all at the University, that is at the Sorbonne. At the school itself the délégués give two conférences per week, in which questions. are asked of the students on the courses they are attending, and exercises are done. During this first year, they emerge from the point of view of the school boy, and in the second year they have greater freedom, beginning with the choice of the subject of their fourth certificate. Apparently however the search for the shortest road to success is one which is not excluded from the École Normale. M. Tannery remarks that a very great number of the Normaliens take Differential Geometry, "besides the interest presented by the teaching of M. Darboux.

This examination takes place at Easter; and it is a satisfaction to the student to be in possession of one of his two certificates so early in the year."

M. Tannery emphasizes however the fact that, although an examination dominates the regime of the École Normale, the aim of the teachers is to instil wide ideals and to accustom the young men from the first to think for themselves, and to consult original papers. Anything of the nature of "cram" is deprecated. Discussion between the students themselves is regarded as one of the most educative factors, and there is much to be gained from the intercourse with the teachers of the school, which, especially in the third year, forms a great part of the intellectual life of the school.

The Department of Public Instruction deals with three different kinds of education : primary, secondary and higher education.

I shall not deal with primary education here, but only with the others. Until 1887 they centralised all the management and the administration of French education. There was one unique university, the University of France. Since then France has been divided into different universities with the following capitals : Paris, Lille, Nancy, Caen, Rennes, Dijon, Besancon, Poitiers, Clermont Ferrand, Lyon, Grenoble, Aix (Marseille), Montpellier, Toulousi, Bordeaux, Alger. In consequence of the treaties with Switzerland, Savoy is placed alone, and constitutes a sub-university, that of Chambéry, At the head of each university there is a rector representing the Government. Chambéry has a Vice-rector, and so has Paris, but in the case of Paris, this is a pure fiction, the Minister of Public Instruction being himself the Rector of the University of Paris.

It is the Parliament which fixes the regulations for the choice of the professors, their promotion, their salaries; the fees to be paid by the students or their families, the syllabus of lectures, etc., etc., in fact everything of any real importance. The universities possesses however a certain autonomy, the fees paid by the students are paid partly to the State and partly to the universities, according to fixed rules. Thus the universities have resources of their own, which they administer themselves, but in reality the expenses which the universities have to meet for constant and urgent needs, are such that they cannot do more than meet them. At the most they are able sometimes to create a new department, or a new chair, or introduce a new laboratory assistant or official. They themselves divide the grants made to them by the State between the different laboratories and departments. In any case the State has the right of control even on these points, and the Department of Public Instruction would certainly not suffer one university to differentiate itself by the creation of new departments, or by dividing its grants in an unusual manner, or in any other way.

I have tried to make you understand this autonomy of the universities by speaking of higher education. It is the same thing in a less degree in secondary education; each Lycée or college possesses a certain liberty of action and of organisation—a little liberty.

Secondary education (I refer only to that of the boys) is carried on in the colleges and Lycées.

To be a professor at a college it is only necessary to have passed the Baccalauréat, but practically almost all the professors have obtained one or even two Licentiates.

To be a professor in a Lycée it is necessary to be an "agrégé." It is however possible to teach in a Lycée without having more than the Licentiate, only in this case the teacher only has the title of "charge de cours" and a correspondingly small salary.

In principle secondary education aims solely at forming cultivated men. The outward and visible sign of it is the Baccalauréat.

To the instruction which corresponds to the aim here mentioned, there has been added a preparatory education for certain great schools, which is given in classes bearing the names of those schools, St. Cyr, Ecole Navale, Ecole Centrale, Ecole des Mines, Institut Agronomique, Ecole Normale (Letters), Ecole Polytechnique est Ecole Normale (Science), (Classes de Mathematique Speciale). (Classe de rhétoriquesupérieure).

These classes are only to be found in certain of the large Lycées. The most important is the class of Special Mathematics. It dominates the scientific teaching of the Lycées. The programme of the Baccalauréat in scientific matters is so arranged that at the moment of taking his Baccalauréat the pupil is ready to receive the instruction of the class of Special Mathematics. The programme of this class is moulded on the requirements for the Entrance Examination to the Polytechnic. As this does not involve Natural History, this subject is hardly taught at all in our Lycées. As it contains a great deal of Mathematics but no arithmetic, there is a great deal of mathematics done at the Lycées, but very little Arithmetic.

There is, I imagine, nothing in other countries at all corresponding to this class of Special Mathematics. Its programme is very interesting, but too extended for most of the pupils to master really thoroughly (although his average of ability is really remarkable). They grasp it for the moment, but they have not really made it their own. It contains, besides Physics, Chemistry and German a considerable portion of Mathematics : Plane and Spherical Trigonometry ; in Algebra, the calculation and the separation of the roots of the Algebraic roots of an equation ; the theory of elimination, symmetric functions. In Differential Calculus, differential coefficients and differentials, maxima and minima of functions of one or more variables. In Integral Calculus, Quadratures, Simple Differential Equations, Analytic Geometry, Descriptive Geometry, Kinematics, Particle Dynamics and the Statics of solid bodies.

This programme is strictly utilitarian. Little by little everything has been dragged in which could subsequently be used by the technicians, and everything which aimed higher has been removed. But this was not always so. Years age the programme did not contain any Mechanics, nor Integral Calculus, but on the other hand the algebraic solution of equations, which has now disappeared, was developed satisfactorily, even so far that the various cases of linear equations were completely explained. At present only the trigonometric solution of these equations is retained in the programme. Continued fractions were also treated and indeterminate forms (l'analyse indeterminee), Analytic Geometry, was then the algebraic study of geometry, now it is reduced to the construction of curves, the reduction of problems on loci and envelopes to equations, and the study of curves and surfaces of the second degree in their reduced forms. The fundamental properties of algebraic curves of the m-th degree, Plucker's formulae, universal curves, the study of a curve in the neighbourhood of one of its points, linear systems of conics or quartics and in particular confocal conics and quartics were studied as examples.

It is evident that those times have left a deep impression on the memories of the professors and examiners, and in practice, although it is the new programme which is taught, a part of the old programme is so also.

It is strange that this programme of Special Mathematics, which weighs so heavily on the whole

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The programmes of the Lycées and the Colleges are the same. The difference consists in this, that the State contributes directly to the upkeep of the Lycées, and the towns to that of the colleges.

of Secondary Education and on the programme of the Baccalauréat established by Parliament, was not itself established by Parliament, but is the work of the Governing Council of the Ecole Polytechnique. No one can understand Secondary Education in France who does not remember that the Ecole Polytechnique is all-powerful. This school is not dependent on the Department of Public Instruction but on the War Office. For a long time it was the only Scientific school in France, it has a glorious past. At the present time it draws its students from the finest youth of France; every year it sends out young men of the most remarkable ability to occupy the first places in the great departments of state. This, joined to a spirit of solidarity which is certainly excessive, and a spirit of tradition which is to be respected, although dangerous, as well as the desire to keep the Ecole Polytechnique unchanged, explain the important role played everywhere by the students of this school.

Some years ago the reform was obtained that the Department of Public Instruction should have complete power as regards the Programmes and that it should publish a maximum programme for the class of Special Mathematics, so that the programmes of the entrance examinations into the Ecole Polytechnique, the Ecole Normale Science, and the Central and Naval schools and the School of mines, should be constrained to be chosen from among the matter given in this official programme. But this was only obtained by (Lebesgue) taking for the official programme that chosen by the Ecole Polytechnique.

The programmes, though certainly open to criticism are in general very good. This may seem strange seeing the way in which they are established. The council of the Ecole Polytechnique contains among its members, one or two persons (school masters) who are really competent for their task, although these are almost lost among the generals, mining engineers, civil engineers (ponts et chaussees), etc., (et des tabacs ?). And all these people have their own ideas and stick to them. But on the one hand, these gentlemen are intelligent and have the sense to ask advice of those of their friends whom they regard as competent, and on the other hand, the spirit of tradition, already referred to, leads them always to go back to what was done at the Ecole Polytechnique in their own days. In consequence it is but slowly that modifications are introduced into the original programme drawn up by Lagrange, Laplace, Carnot, Monge, and Lacroix for the mathematics. And what modifications there are have been made under the influence of successive professors of mathematics, among whom we may note Cauchy, Poisson, Fourier, Ampere, Arago, Liouville, Hermite, Bertrand, Poincare, Jordan.

I lay stress on this : it is a characteristic example of what happens in France very often. An organisation succeeds well in practice owing to particular circumstances, but it would be imprudent to conclude that the same organisation established in different surroundings would give the same results. In particular the French organisations leave room for injustice and violation of the rights of individuals. But in reality there is very little of this.

The programmes established by Parliament are in reality set up by the Department of Public Instruction. For instance the Department appoints a commission to draw up a programme for the Baccalauréat. This commission includes high functionaries belonging to the Department, political persons, financiers, merchants, manufacturers, delegates from various other Departments, and a certain number of competent persons, schoolmasters and learned men. These latter form a very small minority, but the complete commission only meets once or twice ; on those occasions there will be some fine speeches on generalities and some valuable and profound reports will be presented, but in the end there will always be the appointment of a Subcommittee which will include all the competent people, and in particular at the present date, you will always find mathematics represented by Darboux, Appell and Picard. The great commission will ratify the programme drawn up by their subcommittee, and the seal of Parliament will be finally affixed.

The promotion of the professors at the secondary schools is regulated by a commission composed of the Director of Secondary Education (the highest functionary in the Education Department), the Vice-Rector of the University of Paris, the Director and Sub-Director of the Ecolé Normale, and three or four other persons, Rectors and Inspectors General. The credentials to be examined are the reports of the directors of the secondary schools—Provisors of the Lycées and Principals of the Colleges,—the inspection reports of the Rectors and Inspectors, who are bound to visit once at least each year every class and every Master. Among such a great number of people personal relations and political influence have free play. Hence it is not uncommon to find so and so a year in advance of some one else. But on the average nine masters out of ten only owe their advancement to their own merits.

We take next Higher Education, which is of most importance for our present purpose, and we consider the Faculties of Science and Letters, the others (Law, Medicine), having a quite different organisation and aim.

The aim of Higher Education is the same as that of Secondary Education. It aims at making people more cultivated in one of the branches of human learning, that is all. The studies receive the stamp of three diplomas:

(1) The Baccalauréat, which corresponds to the work done before entering the University;

(2) The Licentiate, the work for which is done at the University;

(3) The Doctorate, which marks individual original work.

Till about twenty years ago there was a Licentiate in Letters, a Licentiate in Science (Mathematics) and one in Science (Physics and Chemistry), as well as a Licentiate in Natural Science. The Licentiate in Letters involved one or two examinations on

Modern Languages in branches chosen by the candidate according to the direction in which he had specialised, Grammar, Philosophy or History.

This scheme of the Licence es Letters has never varied, but what is now called the Licence es Sciences is equivalent to the obtaining of three certificates. This was instituted to permit a greater freedom of choice in the subjects of study at the University, to give more elasticity, and more variety. In principle, the Faculties proposed the programmes of the Examinations for the certificates to the Universities, who presented them to the Department of Public Instruction. As for these certificates, which are intended to cover the same matter as the ancient Licentiates, the programmes are very nearly uniform, and the Department pays attention to the maintenance of this uniformity. But alongside the instruction which developed itself following the programmes of the Licentiate, properly speaking, there were certain non-obligatory courses which were usually taken by the students. Thus some twenty years back there were for the Licentiate, which took in general at least two years, courses of lectures and Conferences by Painlevè and Raffy on Analysis, by Koenigs, Appell and Puiseux on Kinematics, and Mechanics, and Wolf on Astronomy, all fundamental and in addition accessory courses, voluntary one may say, by Hermite on Gamma and Beta Functions, Picard on Algebraic Functions, Darboux on Geometry, Poincaré on Mathematical Physics, Boussinesq on Hydraulics and Tisserand on Celestial Mechanics. No one attended all these lectures, but it was considered morally necessary to attend some of them.

The professors who gave such accessory courses, the subjects of which varied from year to year, were afraid that they might have no auditors which was not altogether without foundation, and on the other hand they wished to take occasion to accord to their auditors a diploma showing that they had gone through the corresponding course of study. They accordingly each demanded the institution of a certificate, and thus it came to pass that at Paris there was a Certificate of Higher Algebra and Analysis, a Certificate of Higher Geometry, one of Higher Astronomy, one of Mathematical Physics, one of Mechanics, both theoretical and experimental, and one of Celestial Mechanics. The programmes of these certificates are strange ; that of Higher Algebra and Analysis consists in these simple words: —"the professor's course of lectures", and the same for the others. Yet such a course was of two hours a week during one semester, without a single exercise set and done.

Encouraged by this example, the provincial professors also desired each to have his certificate, and numbers of certificates, with extremely vague programmes, given on examinations still more vague, began to spring up in all the Universities. Some of these certificates soon became celebrated among the students ; you obtained them without fail, provide you attended regularly one hour a week at the professor's lectures. Other professors prepared their students for two separate certificates by means of a single course of lectures; for example General Zoology and General Embryology. It became possible too by means of the same scientific paraphernalia to obtain three certificates thus the same programme, or nearly the same, permitted a X man to obtain at a certificate for Industrial Physics, at Y one for Applied Physics and at Z one for Industrial Electricity; in this way he got his Licentiate!

It is thus evident that the introduction of the system of certificates, though good from many points of view, led quickly to serious abuses and was the occasion of a decrease in the real value of the title of Licencié.

Why did the professors act in this way, and what advantages did they gain? Nothing definite, except that of drawing pupils to their classes. Now this was not remunerative for them, and only gave them extra work, for the professor does not receive any part of the fees paid by his students. But it made a favourable impression on the Dean of the Faculty and the Rector of the University; the professor might hope to be favoured when it came to dividing up the government grants in the share assigned for his teaching, he might hope for more rapid promotion, and perhaps soon to receive a decoration.

But there is another thing, the rivalry between professors. The examiners for the Licentiate formerly consisted of three professors or maitres de conference of the Faculty; each examined on his speciality, or on the part of his lectures which came into consideration. This was not a regulation, but it was what happened, with an exception to be mentioned later. The examiners for the certificates are also three in number, professors or maitres de conferences, the president of the committee so formed is a professor. But as it is an examination on a restricted programme, on a single course of lecture sometimes, it came about that one professor was obliged to ask questions on the lectures of one of his colleagues. And that colleague was not always pleased that he should do so. Add to this that X often objected to being on the board of examiners with Y, unless X was the president, whether Y had given the course of lectures or not; but if X was president, Y would seem to be himself judged by X.

Then the Dean in accordance with the principal professor arranged that the board should be composed of the principal professor himself, his maitre de conference and one of his colleagues, or the Dean, who was however requested not to come and whose duties were reduced to signing the documents afterwards. Sometimes at Paris (in the case of Picard, for example, or Boussinesq), the examination was decided effectively by a single person.

At Paris the older professor who gave special courses did not examine on the subject of their teaching, because it was not expressly included in the programme of the Licentiate. But these gentlemen were the presidents of the boards of examiners for the Licentiate, and the students thought the president would be better inclined towards them if they had attended his lectures- And in point of fact the president of the board was often pleased when recognising one of his auditors in the candidate, he was able to question him on his own lectures. Hence the fear of pupils failing to appear at lectures under the system of certificates.

To remedy these inconveniences at least in part it was decided to distinguish between Licentiate and Licentiate; there was to be the Licentiate as title which might, so to speak, be printed on one's visiting cards, and the Licentiate which should really count, called the Licentiate for Teaching. This second Licentiate is given for a group of three definite certificates, for example the Licentiate in Mathematical Science requires the certificate of Differential and Integral Calculus, the certificate of Mechanics and a third certificate of the type of Mathematics, left to the choice of the candidate, or General Physics.

(There is a very influential body in France which would like to see all these phantom certificates abolished, and in place of them a regulation to be introduced requiring that in order to obtain a certificate a student must have followed besides his compulsory course, two other course of the same type of science, (courses accordingly preparing for no certificate) being rewarded with no title, diligence being its own reward).

The visiting card Licentiate is not without its dangers. A gentleman had gained such a title; he knows nothing all the same, but he becomes a deputy, for example, and this Licentiate seems to his colleagues to render him competent on every subject connected with education, And, thanks to the effrontery of the politician, such a man on a commission may successfully resist Darboux on a question of mathematical teaching.

The licentiate for teaching is demanded from every schoolmaster in order that he may have the position and stipend of a "professor licencie."

It is also demanded for the Doctor's degree. This requires a viva voce examination, in which the candidate has to explain an original thesis which he has written, and is interrogated on a subject chosen by the Faculty, called the subject of the second Thesis. The board of examiners composed of three professors or maitres de conferences of the Faculty, a professor presiding. Almost all the good theses are presented at Paris and not in the provinces; but there are exceptions.

This is the degree of Docteur d' Etat (state doctor). Some years ago, struck with the inconvenience there was in making the obtaining of a Doctor's degree so difficult, particularly in the case of foreign students, when Germany, for example, was distributing this title so widely and so easily, it was decided to create the Docteur d' Universite (Doctor of the university), the tests being the same but without demanding the Licentiate of the University, but only a Licentiate of some sort. This required a less long residence at a French University. Unfortunately the opinion has been spread that this Doctorate is altogether inferior in quality to the other, because certain of these inferior merit have been allowed to pass. In reality there are very good theses for the Doctorate of the University, and there are sometimes quite bad ones for the State Doctorate.

Finally there has been a third kind of Doctorate instituted. The Board of Examiners is as before; so are the tests, only no preliminary degree is required, and the original work may reduce to an original composition without new results. This diploma is called The Diplome Superieur d'etudes (Mathematics, Physics, etc.), The first of these diplomas was created by the Faculty of Arts for the study of history. Besides these courses or study and the preceding examinations, there have been a quantity of others instituted, to which reference will be made subsequently. These courses and examinations are peculiar to the Faculty of Science, except those relating to the Agrégation, which have their analogues in the Arts Faculty. We will now consider this latter examination.

The agrégation is a competitive examination, what is called in France a "concours" not an "examen". It was instituted by the Board of Secondary Education, so as to recruit the best schoolmasters for the secondary schools. The degrees required are groups of certificates or higher diplomas constituting in general one licence for teaching, together with one more determinate certificate or a diploma. The candidates for the agrégation are therefore necessarily students with Higher-Education; thus the Board of Secondary Education finds itself in a certain sense, by reason of the Agrégation, controlling that of Higher Education. In general the students pass the Licentiate, and the best among them pass the aggrégation which assures them a place in one of the Lycées and in consequence a proper stipend, even if a modest one. And some years later the best of these go in for their Doctorate. This is the usual practice, but of course there are exceptions.

The tests for the Agrégation consist of written examinations to control the knowledge of the candidate, oral tests to control his pedagogic qualities, and practical examinations to control his technical capacity. For Mathematics the examinations are :

(1) Written—Four compositions of 7 hours each on set problems on :
    (a) Elementary Geometry.
    (b) Special Mathematics.
    (c) Differential and Integral Calculus.
    (d) Mechanics.
(2) Oral :
    (a) A lesson of preparation for the Baccalauréat.
    (b) A lesson of the class of Special Mathematics.
(3) Practical:
    (a) A piece of calculation.
    (b) A piece of technical drawing.

It may be deemed a reproach to the agrégation that it does not throw light on the originality or want of originality, or on the creative faculty, and yet it proves to be a competitive examination which serves admirably for the selection of the schoolmasters of the Lycées, forming a body of real distinction.

The great Faculties provide conferences as preparation for the Agrégation. At Paris this preparation is given only at the Ecole Normale, considered for the last ten years as an annex to University of Paris.

It is not easy to recruit students for the Faculty of Science. All the pupils who show some taste for Mathematics or Physics aim at the Ecole Polytechnique, some aim at the Ecole Normale, but these are rare, in any case they do not come to the Faculty of Science unless they cannot do otherwise. Now when you consider that the preparation for the Ecole Polytechnique or the Ecole Normale serves also as a preparation for the Ecole Centrale at Paris, Lille or Lyon, the School of Mines at Paris, or at St. Etienne, the Ecole de Ponts et Chaussées and the Agronomic Institute, and the candidates continue their preparation up to the time of their departure for military service, also that often the admission into one of these schools confers certain advantages from a military point of view on the student, while on the other hand the career of a professor is not one offering many attractions in our age of activity, it will not seem strange that University students of Mathematics, Physics and Chemistry are rare. Equally rare are the students of Natural Science, for there is no opening for them in Secondary Education.

Thus when the University came to consider its budget, it saw very soon that something had to be done. Since all the students went to the industrial schools, she proposed to provide industrial courses herself. So they instituted a course on Brewing here, there one on Oenology, there one on electricity according to the tendencies of the region. These courses depend for their value entirely on the individual professor, some are really scientific, others are only to be regarded as preparing foremen. In general they have been a success; it is so agreeable to gain easily a diploma as an expert chemist or industrial electrician. In fact they have been such a success that they have often attracted away the last few students who remained in the really scientific courses. On the other hand the professors which hold these courses, having often a hundred students when the worthy professors of pure Mathematics have perhaps seven or eight, naturally monopolise the attention, the pecuniary profit, the promotion and the rewards. And as it is only human to worship success, this remedy which was introduced so as to Industrialise the Faculties of Science has nearly killed them. They still exist, they possess students, they have colossal budgets, but everything is studied in them except Science.

Among the desiderata which a serious scientist must propose, a considerable place must be accorded to this, the separation of scientific Universities from industrial institutes. Let these latter be created, and connected with the Faculties of Science. But let the connections be other than those of diplomas, or connected with the budget. Let everything be avoided which can create confusion or assimilation between this and that. Let us not permit professors who might be men of learning to sink professors of technical institutes (professeure d'arts et metiers).

Simultaneously, or nearly so, the schools and Faculties of Medicine underwent a process of reorganisation. The same movement which tends to relegate pure science to the second place and put to the applications first, has conducted the doctors, or certain doctors, to seek to free themselves from the criticism of pure science. Certainly it was said that the change was intended to give to the future doctors a solid basis of scientific instruction, but really it gave occasion to cease altogether to talk about a scientific education. Let us pass on.

It was therefore decided to replace the old First Year in Medicine by a year of study in the Faculty of Science, the students taking Physics, Chemistry, and Natural History; hence it was familiarly called P. C. N. This brought a number of students, but only passing students; it was nothing but imposing a class of secondary instruction on the Faculty.

It was evidently this idea of introducting a class of secondary instruction into the Faculty which has been realised by the creation in the Faculty of Science of a course on General Mathematics with the programme (utilitarian) of the class of Special Mathematics at the secondary schools. It was under the influence of Appell that this creation took place. Appell desired that the classes of Special Mathematics should gradually be transferred completely to the University. His hope was that, after having inveigled the pupils to the Faculty of Science, they would not all of them forget the way there, and some at least would stay. The basis of his idea was on the other hand to obtain the result that, as it is probably desirable should be the case in the Technical Institutes, and as is realised, perhaps too much, in the P. C. N., the scientific preparation should be divided from the technical training. He foresaw the Ecole Polytechnique, the Ecole Centrale, etc. reduced to technical schools, and the preliminary scientific training given to the Faculty of Science.

But the Ecole Polytechnique was on the watch, and we are still only at the first stage of the creation of a course of General Mathematics of a. low grade. This course, because of a low grade, has attracted a large number of students and it has added to the harm already done to higher studies; firstly because the level of the other studies has been sunk to put them within the reach of students who have passed through the course of General Mathematics; secondly because in the smaller Universities it is the same people who give this elementary instruction and the higher teaching. As they have no students except for the former, they end by losing all interest in higher teaching.

The influx of students is particularly felt at the Ecole Normale, where it is quite recent. The Ecole Normale dates from the Revolution. It was closed after three or four years, but was re-opened after the Restoration chiefly to bring the spirits of those whose duty it was to speak to the youth of the country into line, to discipline them, and to superintend their development. But this view of the Ecole, which was that of the Government, has never been that of the students. The Ecole Normale has been on the contrary always a hotbed of Liberalism.

No matter; the Ecole existed and from the point of view of Science its existence was of the capital importance. Its renown, its splendour, attracted to Education and to Pure Science some of the better pupils of the Class of Special Mathematics who would almost certainly have been lost to Science if they had gone to the Ecole Polytechnique.

But recent transformations in its internal organisation and its mode of recruitment have been falsely interpreted, and made people think that the ancient Ecole Normale no longer existed as they had known it. Moreover the youth of the present day, in love with an active life, with sport, with action, consider that to pass one's life with the chalk in one's hand teaching at the black board is not an attractive future when it is possible to pass one's life in the open air, to be, for example, an officer in the army. Again the young men of the present day, sure of themselves, know the advantages of the different positions. If you go through the Ecole Polytechnique, you are certain at least of becoming an officer, you have a free, open air life, hygienic, the pay is now reasonable, you have a position in the social world, you can marry advantageously. Naturally if you come out top, or near the top, you can count on a great deal more.

If you go through the Ecole Normale it is necessary to obtain the Agréegation at leaving, or your career will be a poor one, and even in that case the career open to you is decent, but modest. One per cent perhaps, of the students obtain really a, brilliant post.

The entries into the Ecole have considerably decreased on the average. It is wonderful that so many distinguished people still come out of the Ecole Normale. Formerly the competition for entry into the Ecole Normale was distinct from that for the scholarships (bourses de licence), although, these two examinations, like Entrance Examination for the Ecole Polytechnique, have for programme that of the Class of Special Mathematics, except as regards Natural History. Now there is only one real competition. The Board of Examiners consists of one or two provincial professors or M. de C. and of certain professors or M. de C. of Paris, in general those who are engaged at the school; those who come out at the head of the list receive the scholarships (bourses de licence) at the University of Paris, that is to say at the Ecole Normale, the others receive such scholarships in the provincial universities. This modification has thrown a certain discredit on the Ecole Normale, but without any real justification. To be a scholar (Boursier de licence) at the University of Paris means free quarters, if desired, and commons at the Ecole Normale, or instead a certain sum of money for board and lodging, and free access to the courses and conférences at the Ecole and at the Sorbonne, with the use of the library, etc. In the provinces the Boursier receives a certain sum of money. Moreover both at Paris and in the provinces the Boursiers dispensed from all fees.

The students, whether French or foreign,* have in fact to pay fees for inscription, giving the right to attend the lectures (though in practice it is possible to attend without paying) and permitting the University to control the length of time the student has studied. A certain time of study is demanded before each examination. Moreover there are the fees for examinations, for the use of the library, for practical work, laboratories, etc.

These fees furnish funds which go partly to the state and partly to the University. In no case, do any of these funds go to recompense a professor who has a great many students. There is no capitation fee paid to the professor.

The teaching in the Faculty of Science is given by Professors and Maitres de Conférences. There are also "Chargés de Course", Assistant Professors and "Chargés de Conférence", but these different titles correspond to such small differences without any importance that we may pass them over.

To be appointed a Maitre de Conférence (M. de C.) it is sufficient to have obtained the Licentiate; in practice however the Doctorate is demanded in the Faculty of Science; in the Faculty of Arts, where the thesis often takes ten years work, they often appoint their M. de C.'s before the publication of the thesis, which is already known in part.

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*For the majority of courses the Baccalaureat, of a foreign equivalent is demanded.

To become a Professor is necessary to be a Doctor, and to have been an M. de C. for two years at least.

The Minister of Education is free to appoint as M. de C. whoever he pleases, subject to the restriction mentioned above. He may appoint as many M. de C. as he pleases in each Faculty. In fact the number of M. de Cs.' which is very variable from one Faculty to another, is rarely modified. Posts have been created in towns where the need for them had made itself felt; but it is practically unread of for the Minister to create a post for the mere benefit of some persons who happended to be in his favour. The nominations are on the whole well made and almost always go to the person with the greatest merit : this is because, without been obliged to do so, the Minister always asks and follows the advice of the "Permanent Commission". This Commission, appointed by the Minister, is intended for his enlightenment. It is composed of whoever he pleases; at the departure of one Minister, his successor can appoint whoever he likes. But the tradition is anyone appointed to this Commission remains on it all his life. It is composed of the three directors of education, the Vice-Rector of the University of Paris and certain learned men or school masters. It is really on the whole very impartial and just. It is however to be remarked that, in each particular case, it only contains two or three members really competent to judge, sometimes only one, sometimes none at all (at the present moment there is no Zoologist on the Commission, and no Botanist). Hence the demand has recently been made that the Commission should always contain at least two members to each subject.

This Commission draws up every year in order of merit the list of possible M. de Cs.' in each subject, and it is from this list that the Minister makes the nominations.

A Maitré de Conférence is only appointed for one year. Every year he is obliged to be reappointed. If the war continues it is possible that, for reasons of economy, some of the M. de Cs.' will not be reappointed next academic year. There are some M. de Cs.' who are not subject to reappointment, these are the Assistant Professors.

The nomination of a titular Professor however is for life and is made by a decree of the President of the Republic. The posts of the professors are called Chairs, they are created by the President of the Republic on a vote of Parliament if a grant is necessary. There are few new chairs created, perhaps one in three years.

When a chair is vacant, it happens usually that the Department of Public Instruction appoints a "Chargé de Cours", that is to say an M. de C. who takes the place of the titular professor until the appointment is made. The new Professor is appointed by the Department, following the advice of the Section Permanente, but they appoint whoever they choose. The appointment may be made with or without the nomination of a Chargé de Cours.

The Faculty first, then the University, demands. that the Minister should declare the Chair vacant. When this has been done, the candidates have 20 days to send in their credentials and their publications. The Faculty draws up a report, and writes down in order the two names which it considers the most distinguished. The University then votes in its turn and always ratifies the two names. These two names are then sent up to the Section Permanente, which itself gives two names, not always the same as the others. And the Minister, to whom these two lists are sent up, proposes one name to the President of the republic. The Minister always choose the first name on the two lists if it is the same, if not, it is almost always the first name on the list of the Permanent Commission which is chosen.

In point of fact the Faculty is not always impartial. If a Chargé de Cours has been appointed, there is no difficulty, it is his name which stands first. But if there has been no such nomination, the Faculty may be quite badly informed and is also naturally partial. It may be badly informed, for instance, if there is only one Professor of Zoology, when that Chair falls vacant; there will then be no one on the Faculty competent to judge and classify the candidates. Such a case occurs not frequently, and almost always there are at most two competent judges on the Faculty. The Faculty may be partial for a different reason; if an M. de C. already on the Faculty is a candidate, the professors know him personally, and they do not know the other candidates, so it will often be he who is designated by the Faculty, whether he is the candidate of greatest merit or not.

The promotion of the M. de C.' s and the Professors is regulated also by the Section Permanente or at least according to its suggestions.

There are sometimes M. de C. who are entirely responsible for a subject, they are then called sometime "Chargé de Cours" or better "Chargé d'un Cours complementaire de...."; there is a great variability in the titles used. But nearly all the M. de C's are attached to a Chair occupied by a Professor. Then the Professor charges the M. de C. with a part of the subject, for example, Goursat is the Professor of Differential and Integral Calculus, and Lebesgue is his M. de C. Goursat takes the greater part of the course, and he asks Lebesgue to speak of the applications and to take charge of the exercises of the students. Beyond this, the Professor has no right of control over the M. de C. At the outside, he might, if he thought that the M. de C. performed his part badly, point this out to the Dean of the Faculty who might speak about it at the Department of Public Instruction; in this case some men of distinction in the subject might be sent by the department to inspect the lectures of the M. de C., to draw up a report. Such a thing happens very really, once in the last fourteen years, or thereabouts.

In principle, the professor has no authority over the M. de C. Often if the professor in autocratic and the M. de C. is juvenile and patient, the former does arrogate to himself the right of control; but these never reach very far, and are more or less on the surface. On the other hand in laboratory teaching the Professor may exercise a real pressure on the M. de C. Here the professor is sole master of the funds at the disposal of the laboratory, so that the M. de C. is entirely dependent on him for the expenses of the laboratory contingent on his teaching as well as those demand by the M. de C.'s own private work. This is a thing to be regretted; the consequences however have not in general been very serious, for, where a professor went too far in this respect, it has almost always happened that the University gave a personal grant to the M. de C. reducing by a corresponding amount the grant made to the professor. The "Chargés de Cours", although not titular professors, control the funds of the laboratory under their care.

Neither does the State itself exercise any pressure on the teaching of the M. de C.'s or professors. It has never intervened except when the teaching really occasioned a scandal. The only thing possible legally (unless the facts fall within the scope of the common law) is to suspend the course of lectures, which does not involve the suspension of the salary. If it is a M. de C., he might not be reappointed, but whether this has been ever done or not I do not know. If it is a titular professor there is nothing to be done legally. It was by a violation of the laws that at the time of the Coup d'Etat of 1872 some of the professors were dismissed.

In principle the professors are independent of each other. In practice certain of them, by reason of their age and distinction have a greater influence on their colleagues and are more listened to, than others on the councils of the University. It happens that such men have their own circle of adherents. The report here given shows what are the principal points on which it is desirable that changes should be brought forward.

(1) Pure Science requires to be protected;

(2) The system of certificates must not be permitted to reduce to a skeleton the teaching for the Licentiate. Whether the courses without certificates are multiplied to the extent which was the case some years ago, or not, the whole circle of scientific knowledge must be represented. For example it should not be possible that Algebraic Geometry and the Theory of Forms should not have been taught in France for the last 15 years. Among these courses there ought to be some of a special character, rather pedagogique, and these should be obligatory; we will go nearer into this, point.

As will have been seen the Licentiate for Teaching is given at the end of studies of a scientific character bearing on matters which the masters at the colleges do not have to teach. On the other hand the Agrégés have to pass certain pedagogic and technical tests, corresponding to which there is nothing in the Licentiate. Without demanding the insertion of such tests, it might be well that the Faculty of Science should provide certain courses of lectures for the purpose of widening the horizon of the future schoolmasters as to the subjects which they will have to teach. For example, we may take Klein's lectures first and then geometrical constructions like those in Enriques's book, questions concerning elementary Mathematics, and so forth.

These voluntary courses on various subjects which make greater demands on the originality and require more work than the ordinary courses, would give an opportunity for original work; it would be well to assign them to the young professors in preference to the others. On the other hand, the older professors, who have had more time to reflect on Science as a whole, might be entrusted with fundamental courses from general points of view, requiring less originality but a longer experience and deeper thought.

At present in all but two or three of the French Universities, the staff is too restricted for the organisation of such teaching, and they could not be instituted without suppressing two or three phantom Faculties, such as that of Poitiers. At the great universities, at Paris in particular, where there is more than a mere skeleton of a Faculty, it is the old professors who give the courses demanding originality and the young ones who have to teach the subjects as old as the hills. For example Lebesgue, whose work has revolutionised modern mathematics, has not yet had the opportunity of lecturing on his own work, though he has been for some years at Paris.

It might be well also to introduce the principle of External Examiners, or at any rate to demand the presence of members from the outside at the examinations for the Licentiate, (as in the case of the Examens de naturité at Geneva, where representatives of the Polytechnic at Zürich are required to be present). This might prevent the standard of the diplomas falling too low. Thus here is a story which I knew at first hand. In a small provincial French university, the young professor had a class of two students only. One of these was a man of 42 years of age, with three children. He entered for the certificate of Mechanics. This certificate would make all the difference to him in his career as a schoolmaster. He was personally acquainted with the professor. Could the Professor refuse him the certificate of his own unaided responsibility?

As the procedure in France in the matter of the importance of examinations alone of all European countries at all resembles that of England, it is of great interest to notice how essentially different the actual examinations are from those of England in character. We cannot do better than consider in this connection the Entrance Examinations into the Ecole Polytechnique, which practically determine all the corresponding examinations elsewhere. Through this institution, which is of more than University rank, all the best mathematicians of France have passed. The age of entry may be anything from 17 to 21, and the candidate must have already obtained the Baccalauréat, and passed through the class of Mathématiques Spéciales. Practically all the best Frenchmen pass through it, if they are able to reach the standard. The course lasts two years, and accordingly corresponds to the first two years at the University. The course of Mathematics corresponds to that for the Licentiate at the University. The general public is interested in the names of the candidates who occupy the first places on entry and on leaving the school. Thus it is recorded that Hademard, the well-known French mathematician, Member de l' Institute, still holds the record, which he made in 1884 of the greatest percentage of marks at the Entrance Examination. The analogy with the Mathematical Tripos at Cambridge is not far to seek. But how different the character the examination, and the mode in which it is carried out !

Take the examinations for entrance into the school. It comprises three sets of tests, two oral, and one written. Of the oral examinations, the first is so important that it settles of itself the question whether the candidate is to be examined to the other two examinations at all. It is also necessary for admission to the remaining examinations that the candidate should first pass satisfactory written examinations in certain of the subjects, *viz.* Mathematics, Physics, Chemistry and a practical exercises in Descriptive Geometry.

In the actual examination to which the candidates who have satisfied these first requirements are admitted, 82/121 of the total number of marks are given for the oral part of the examination, and only 39/121 for the written part. In Mathematics 50/121 are given for the oral, and 17/121 for the written work. Physics obtains 14/121 for the oral, and 6/121 for the written examination; Chemistry 7/121 for the oral and 3/121 for the written examination. In German the examination is entirely oral, and 8/121 of the marks are allotted to it, as against 6/121 for French composition, of course written.

The oral Mathematical Examination is conducted by two examiners, each of whom allots half of the marks. Drawing obtains 7/121, and Physical Qualifications, Fencing, Gymnastics and Riding 3/121. Some extra marks may be obtained by a voluntary examination in translation from French into a modern language other than German, amounting to approximately 1/121 for each language, except that in English a candidate who has passed a sufficiently good written examination may claim to be examined orally, in which case he can obtain 2/121. The fact that so much importance is given to German is due to the circumstance that since 1870, the army, under whose control the Ecole Polytechnique if placed has had to fear the aggression from Germany which has now actually taken place. This preference for German has had the result that French mathematicians have had practically no difficulty in reading German original papers, whereas their knowledge of English has frequently been insufficient for such a purpose.

At the Ecole Polytechnique the programme in Mathematics includes Analysis, Geometry, Mechanics, Astronomy and Geodesy.

In Analysis of the first year includes Reduction programme of the Elliptic and Hyperelliptic Integrals, the precise definition of Definite Integral, including the theory of Definite Integrals whose limits are infinite, or whose integrands are discontinuous. Integration of total Differentials. Application to the Geometry of Curves and Surfaces, including Conformal Representation, Multiple Integrals and application Green's and Stoke's Theorems, together with their applications in a vectorial field, not necessarily continuous, but also such as may occur in Electricity and Magnetism. Potential fields, and Solenoidal fields are specially studied. The asymptotic value of T (n) is led upto on account of its importance in the Theorem of Bernouilli and Probabilities, and in that of the discussion of errors in the course on Astronomy. Finally, Fourier Series is tolerably completely discussed, so far at least as Dirichlet's treatment of its convergence and its convergence and its applications to Physics.

In the second year the attraction of a solid body and that of a simple or double layer, the law of attraction being that of Newton, is discussed, and the classical formulae of Poison and Gauss are obtained. The discontinuities of the differential co-efficients of the Potential function, and of that function itself are also considered, and in particular the attraction of a magnet and the corresponding potential, as well as Magnetic Induction are treated. In the same connection Spherical Harmonics, Dirichlet's Principle, and their applications to electricity, such as that constituted by the theorem called by the French the Charles-Green Theorem and Faraday's screens are studied. But in this course the object is to prepare for and render intelligible the course on Physics, and not to pass out of the region of Analysis, which is naturally concerned only with the formulae and their rigourous discussion. Analytic Functions are next considered, in consequence of the fundamental role planned by them 'in Differential Equations, whose discussion forms the basis of Natural Philosophy. The treatment however is an abbreviated one, as confined to general theorems on uniform functions and their integrals, Taylor's Series, Power Series, certain analytical applications, and a few notions with regard to Elliptic functions. The point of view adopted throughout is that of Cauchy.

Differential Equations are then considered, commencing with the existence Theorems. The linear systems, so important in the applications being especially considered. To the ordinary cases of integration, already known to the pupils, that of the equation of Riccati is added. Some examples are given in which equations of higher order are reducible to the first, in their relation to certain problems furnished by Geometry and Mechanics. Integration by means of definite integrals and by series, are treated at length, including a discussion of Laplace's and Bessel's equations, and Fuchs's results as to the form of the solution in the neighbourhood of a critical point. Partial Differential Equations of the first order are studied in two simple cases only. The general linear equation, and non-linear equation with two independent variables. The equations of the second order are only treated in particular cases; the equation of the telegraphists serves to furnish a reason for explaining 'Riemann's method of integration; that of permits the introduction following Boussinese of the Spherical Potential and so following the propagation of a wave of sound. Certain equations finally borrowed from the Theory of Heat bring out the aid in integration furnished by Mathematical Physics. The Theory of Characteristics, and their connection with the Propagation of Motion are only examined in linear cases. Calculus of Variations forms the last part of the course; necessary conditions only for a maximum or minimum only are indicated, namely the equations due to Euler, Cauchy and Lagrange. The question as to whether the function determined by these equations really corresponds to the problem considered is not discussed. The principles are applied to ordinary questions, such as those of the Brachistochrone, catenary and minimal surfaces.

The examinations on this course. By a deputy of the professor during each of the two years each pupil is examined orally, three times for a quarter of an hour. At the end of each year there is besides a special examiner who examines all the pupils orally, devoting $\frac{3}{4}$ of an hour to each. The marks awarded by the later count very high in the sum-total.

Besides these oral examinations there are three written examinations each year, set by the professor, and looked over either by him or by a deputy.

In Geometry, in which the course is less consequential, but which is especially characteristic of the Ecole Polytechnique, as it contains all which interests an engineer at the present day, namely Perspective, Stereotomy, Nomography and Graphical Statics. The duration of the course is a year and a half, comprising 26 lessons in the first year and 12 in the second as against 65 in Analysis. In the first year conical perspective, the corresponding trace, and direct constructions which are deduced from it lead to the study of the inverse problem, that of Restitution, to which are added certain notions on Photogrammetry, this part of course being terminated by particular cases of perspective, cavalier, axonomatric and isometric. Poncelet's Theory of Projection, excluding imaginaries, is naturally referred to.

The professor next passes to Infinitesimal Geometry, including applications to problems of Geometry and Mechanics, to construction of tangents, points of contact, and centres of curvature. In space the infinitesimal theory of ruled surfaces and developables, properties of the indicatrix, asymptotic lines and lines of curvature and geodesics are encountered once more by the pupil, this time in their geometrical dress.

The Geometry of Kinematics then follows. The study of the movement of a figure in a plane leads to the theory of the instantaneous centre, and Savary's construction for the radius of curvature of the path of a point. The discussion of movement in space leads to the introduction of the focus and characteristic plane, and the theorem of Schoenmann-Maunheim, and gives rise to numerous applications.

Two lessons are devoted to Geometrical Optics, namely to the Theorem of Malus-Dupin and the Wave Surface.

Pure Kinematics is next discussed in connection with the most general movement of a solid, and the composition of translations and rotations and is followed by Applied Kinematics, where the principal mechanisms are passed in review which have been designed to transform a movement, wheel work of all kinds, endless screws, systems of toothed wheels, epicyoloidal trains, rolling curves, joints, eccentrics, bielle, balance, Watts' parallelogram, encliquetages, photographs and inverters. Finally the first year's course is terminated by the study of certain planimetric models, integrating machines and harmonic analysers.

In the second year stereometry in reduced to six lectures, two on sections of wood, four on sections of stone. Six more are devoted to Nomography and Graphical Statics.

The applications consist in perspective drawing, stereometric drawing and the drawing of organs of transmission.

The examinations are three in number in the first year and two in the second, all viva voce. General examination takes place at the end of each of the two parts of the course. It should be remarked that whereas the course of Analysis is entrusted to two professors, each of whom takes the whole course, and has his own pupils throughout the two years, there is only one professor of Geometry for all the pupils. The course in Mechanics is given by two professors, each of whom keeps his pupils throughout the two years, the course, each year consisting of 38 lessons. The programme differs from ordinary courses in Rational Mechanics in consequence of large additions of a technical character, dealing with machines of every kind, with Elasticity, Hydraulics and the Resistance of materials. Moreover throughout each abstract theory is always illuminated by problems borrowed from experience or practice, purely fancy problems being excluded.

In the first year, pupils spend a short time in revising the theory of vectors, in particular the composition movements, and by way of example, the movement of the Earth in space is considered, and that of a solid body moving on a fixed body; in this connexion the Theorem of Coriolis and its applications is considered. The Dynamics of a Particle is completed, and includes the consideration of the ballistic curve in a resisting medium and the problem of the coil or cartridge (? fusée) Vibrations of a spring the movement of particle on the Earth's surface, deviation of projectiles, Foucault's and spherical pendulum.

The treatment of the Statics of a system starts with the Principle of Virtual Work, and Lagrange's method of reducing the problem to the case when the weight is the only force. These methods are applied to the equilibrium of heavy systems, brawbridges, balances, and other weighing machines, also to solids which are free or constrained, screws in their holes, locomotives on their springs of suspension, aeroplanes, articulated systems, cranes; details being fully considered. The equilibrium of strings or chains gives an opportunity for studying suspension bridges, funicular polygons, aerial carriers. A great deal of attention is bestowed on general dynamical principles, that of Hamilton, d'Alembert and Lagranges Equations, these are applied to numerous problems, and the extension to the case of percussions is explained. Kinetic, potential and total energy are considered, and numerous examples of fire arms in particular. The Theory of Small Movements, so important in Physics, is considered at length. All the above theories are then applied to simple machines, friction being taken into consideration, the inclined plane, the wedge, and press, brakes, etc., etc. Friction in wheel work, the stiffness of chords and its friction on a fixed cylinder, etc., etc., etc.

In the second year the Dynamics of Solid bodies is studied, beginning with moments of inertia. Rotation round a point is studied in detail by the methods of Lagrange, Resal and Poinsot. The mono-rail, the gyroscopic effect, the gyroscophs of Foucault with a directrix plane, weighted; spinning tops and the phenomena of precession. Applications of the movement of a free body are made to the hoop, the billiard ball, the heavy cylinder moving on an inclined plane, the transport of heavy weights, the traction of a carriage, the uncoupling of a locomotive. The extension of Dynamics to deformable systems is applied to the movement of the crane, swing, etc. The theory of the Impact, of Solids, and that of Percussions is applied to the shock of a wheel against an obstacle, to the pendulum with sudden variation, etc. Elasticity begins with the geometrical study of Deformation, and ends with the calculation of internal tensions, and the ellipsoid of tensions. The equations which govern the equilibrium of elasticity are given, with special cases, such are that of uniform normal pressure, that of the longitudinal extension of a prism, that of a cylindrical layer, which introduces the principle and the formulae (? frettage). The equilibrium of a spherical layer, of a rotating body, of a twisted cylinder, and the general problem of Saint venant, with a few notisms on simple and complex flexion, and the curve of elasticity. Applications to the propagation of plane waves, especially of luminous waves are given, and the course terminates by a brief account of the resistance of materials, in which examples are taken from the fatigue to flexion, the effort of cutting and of twisting, the fatigue of a boiler and of a fly-wheel. of the flexion of a straight or curved bar, and of an steel spring.

The course then takes in Hydrostatics, including stability, and Hydrodynamics, where Euler's and Helmholz's equations, and the Theory of Vortices, ground swell, chomping of the waves, are considered. It includes also Hydraulics, where beside Bernoulli's Theorem, and the formulae of Navier and Zeuner, the dropping of gases and liquids is considered, to movement of water in a canal and a tube, and the carrying of a solid by a fluid are studied. Finally machines and their principal parts are discussed, not only as regards that their work, but also in detail. Various types of motors, including wheels and hydraulic turbines, steam turbines, combustion and explosive motors, etc.; dynomometers, fly-weels and regulators, transmission by straps, and teledynamic cables. Special consideration is given to the theory of aeroplanes and helicopters (?), including problems of stability, turning and descending as they present themselves.

The examinations are of a similar character to those in Mechanics, with the addition of the Drawing of Machines; actual manipulations are also carried out by the students during this year to enable them to make the necessary measurements.

Under the head of Astronomy and Geodesy a course of 17 lessons is given, but this course includes the Theory of Errors of Observation, including the Method of Least Squares and Quadratic Mean with applications. The Calculus of Probabilities is taught at the same time, the probable error being defined and Bay's Theorem being explained and the Method of Least Squares justified. The course taken at a whole begins with the Astronomy of Position, and finishes with the study of the stars, moon and planets, including the systems of Hipparchus, Tycho Brahe, Copernicus, Kepler's Laws in connexion with Newton's Law of Gravitation, the perturbations of the planets, masses of planets provided with satellites, stability of the solar system, determination of parallaxes, movements of the moon, calculation of eclipses, Newton's and Laplace's theory of the tides, their prediction and calculation, mathematically and mechanically. Precession and Notation are treated in a single lecture and the last lecture is divided to steller and solar astronomy. Each pupil has to undergo two oral examinations as well as a general examination in Astronomy. They are also exercised in the management of the theodolite, the meridian circle, the sextant, and they visit the observatory of Paris.

SWITZERLAND

The Universities of Switzerland are of such recent date that it is not surprising that in many respects, the mathematical departments are not satisfactorily provided with what is needed, and not well organised. In German Switzerland the organisation of the universities is taken ready made from Germany, and many of the mathematical professors are Germans, (on the other hand one Swiss professor, Einstein, has been called to Berlin lately, and the fact that Euler was a Swiss shows that, although a small country, Switzerland ought not to be too modest as to her possibilities of shining in the mathematical world).

The Swiss Universities are not national (Eidgenossisch, Fédéral) but Cantonnal. The Polytechnicum at Zürich is the only national institution of this kind, and this although by the Statutes of 1848 permission was given for the foundation of a national University as well as a Polytechnicum, of which only the latter has been realised, being founded in 1855.

The only ancient universities in Switzerland are that of Basel, founded in 1459 and Geneva founded in 1559, the latter being called the Academic until 1873 when the institution was re-organised, and turned into an University in the modern sense. One of these Universities is of quite recent date and the remaining Swiss Universities, four in number were all founded during the 19th Century, they are at Bern (1834), Freiburg (1895), Lausanne (1890), Neuenburg (1909) and Zürich (1834), Geneva, Lausanne and Neuenburg are French-speaking, Basel, Bern and Zürich German-speaking, and Freiburg bilingual, with a leaning towards French.

Under these heterogeneous circumstances, it is not surprising that there is little unity in the organisation of the mathematical departments in Switzerland. Moreover the students in the mathematical departments are generally rather weak, and take mathematics chiefly or entirely with a view to obtaining posts in the secondary or primary schools, or else, as at Geneva, they are Russians, chiefly Russian Jews, with very little previous training. The difficulties of supply and demand, one party saying there is no supply, the other that of course there is no demand, since there is no supply, have prevented any great ameliorations of the conditions, whether of professors or students. On the other hand the bourgeois class, who have the power, have little comprehension of mathematics, and easily expend there efforts on more congenial subjects, *e.g.* Medicine.

The study of the mathematical departments of modern universities convinces the writer that in order to be satisfactory it must have either (1) an ancient foundation, dating from the time when mathematics was the highest erudition, or (2) be under the control of an autocratic and powerful authority, formed of people of the upper classes. It seems impossible to get the bourgeoisie to understand the importance of a satisfactory mathematical department in a modern university. It is to be noticed that in Swiss, as in both French and-German, Universities there is complete freedom both as regards the professors and the students (Lehr—und Lern—freiheit).

The appointment of professors is made by the Government of the Canton, the Faculty in which the vacancy occurs generally making more or less informal suggestions, or advice being asked from a committee of experts appointed for the purpose. The professor may in some cases be "called" in the German way, and this is usually the case in the German Cantons, or there may be a competition in the French way, as is usually the case for example at Geneva. There is usually a tendency to give the advantage to a local man but in the German Cantons the best mathematical posts are constantly held by Germans and not by Swiss. In Freiburg, which is a Roman Catholic University, the religion will be taken into serious consideration. The appointment is usually for 6 years, after which reappointment is at least formally necessary; except in the older Universities of Basel and Geneva, where the appointment is for life. A certain minimum number of hours lecturing is usually, varying, stipulated for in the terms of each appointment, in the case of ordinary professors from 9 to 12, and in the case of extraordinary professors 4 to 6.

The scheme of teachers is that in use in Germany, Professors, ordinary and extraordinary, and Privat docents, the latter for the most part, as in Germany, unpaid, except in so far as they receive a portion of the fees paid for attendance at their lectures, but sometimes, *e.c.* at Bern receiving a regular stipend from the University. One of the two professors of Mathematics at Geneva has an Assistant, who takes part of the work falling properly to the share of the professor, who is of delicate health, and receives a corresponding remuneration. Otherwise the Swiss mathematical professors do not have Assistants.

The number of mathematical students at a Swiss University is in any case small. All the Swiss Universities are open on equal terms to men and women, Swiss and foreign, in so far as they can present credentials authorising their immatriculation, or even, as hearers, with inferior qualifications. Those who have passed the Examen de Maturité at Geneva are accorded at Geneva a certain reduction of fees, but these are few in number and for the most part women.

The majority of boys who at the College have shown any special aptitude for Mathematics, go on, after their Maturite, to the Polytechnicum at Zurich. This institution, the only Federal institution for Higher Education in Switzerland, as already pointed out, dominates to a great extent, on the pattern of the Ecole Polytechnique at Paris, the Mathematical education of Switzerland. Representatives from this powerful institution are present at the Examen de Maturité in Geneva and elsewhere, with the good consequence that the standard of this Examination is maintained at a high level. It is perhaps due to the same good influence that we find such remarks as the following in the pamphlet issued by the University of Basel for the benefit and advice of the students of Mathematics and Natural Science :—

"It cannot be sufficiently emphasized how important it is at an early date to gain the necessary manual dexterity in Geometrical Drawing for the purposes of Descriptive Geometry".

It is evidently found in Basel, as in other Swiss Universities, that students obtain entrance into the University without a sufficient preliminary training. The Swiss Universities, like the French, have accordingly occupied themselves with a certain amount of Mathematical training which should properly be obtained at the secondary schools, and is in fact provided in the Technical Section of the Colleges. Among the subjects thus taught Descriptive Geometry holds a place. Elementary Differential and Integral Calculus, however have their proper place in the first-year at the University, as in Germany, the French idea of starting the University course with Higher Differential and Integral Calculus not being adopted. The "Ergänzungsvorlesungen" (supplementary lectures) at Basel are accordingly on (a Algebraic Analysis and b) Descriptive Geometry, and do not include the Calculus. The programme of Basel University is interesting chiefly as showing how the proper ideals are maintained there, and the little volume above referred to is to be regarded from this point of view. The University instruction is divided into three headings, as follows :

(a) Preliminary lectures.

1. Analytic Geometry of the Plane and Space, (4 hours).
2. Differential and Integral Calculus I and II, (4 hours, 2 Semesters).
3. Exercises on the Calculus, I and II, (4 hours, 2 Semesters).

(b) Courses of Lectures.

1. Algebra.
2. Theory of Functions.
3. Elliptic Functions.
4. Ordinary Differential Equations.
5. Partial Differential Equations.
6. Linear Differential Equations used in Mathematical Physics.
7. Projective Geometry.
8. Theory of Surfaces.

The student is reminded, without mincing matters, that unless he has mastered the Preliminary lectures it is useless for him to attend these course, which are intended to give him an introduction into the most important branches of Mathematics.

(c) Special Lectures.

These are variable. Mention is made only in general terms of Elementary Geometry and Theory of Numbers, for their application in School teaching, Calculus of Variations for Mathematical Physics. Also attention is called in the little book above mentioned to the close connection between Mathematics and the modern study of the consciousness

(Erkenntnistheorie), and the importance of the latter in understanding the History of Mathematics. I quote all this to show that the standard of Basel University is one of deep thought and not the mere superficiality of an institution working merely for examination purposes.

A glance at the list of the lectures given at Zürich and Bern shows also that, whatever may be the actual results in practice, the ideals of these Universities are those of the best German school.

At Geneva, as already mentioned, there are certain difficulties of a local nature, and the level of the Mathematical school is not high. It does not indeed attempt to go far. Yet, apart from the really cultivated and inspiring teaching of the professor of Analysis and Mechanics, a man who, in spite of poor health, has pursued a line of original thought of his own, and has a remarkable power of exposition, there is that at the University of Geneva which for us requires mention. It is the attitude of the professors of subjects outside Mathematics towards that subject. Not only the distinguished professor of Experimental Physics strongly recommends the study of Mathematics beyond those mere elements considered by some of his less eminent colleagues in English speaking countries as sufficient, but the professor of Botany, one of Geneva's most remarkable men, insists on the value of Mathematics for his subject. I have myself heard him explain to his large class filling the great theatre of the University, that it was a mistake to suppose that a thing which was expressed mathematically was difficult. What is really difficult to understand, and often incomprehensible, is the elaborate explanation of the unmathematical scientist. A thing which is explained mathematically is clear, and anyone can understand it who takes the trouble. One of the taughest objects of the serious scientist is to master the machinery of mathematics and use it in his own branch. In this connexion I cannot refrain from referring to the valuable mathematical researches on Geotrotism lately published by two Swiss botanists.

The attitude of the brother scientists at Geneva towards Mathematics is so encouraging that I cannot but believe we shall soon see an improvement in the staff and equipment of the Mathematical Department itself. In the meanwhile I should like to signalise the existence, although somewhat embryonic, of a student's reading room and library for Mathematics outside the University Library. The latter has a good collection of mathematical periodicals and dissertations, and a certain number of older works, but is poorly stocked with modern mathematical literature. In my own stay at Geneva I have not only found most of the facilities in the shape of books that I required, but an intellectual atmosphere of dignified sympathy and stimulus, which I should be glad to graft into the great Eastern University with which I have the honour to be connected.

In detail I have nothing more particular to say about the remaining Universities of Switzerland, except to point out that the general attitude towrads

Mathematics is friendly and that mathematical research is not only welcomed, but regarded as worthy of reward in the shape of professorships. I well remember the sympathetic tone in which a young mathematician, now extraordinary professor in his native town of Freiburg, was welcomed as a Privat docent at Geneva by his older colleagues both in his own subject and in others. His already published work, and the fact that he had spent a year of study at Göttingen were pointed at as promising facts, pointing to a career in the future.

The institutions of the Seminar and Proseminar have been borrowed by Switzerland from Germany; to be noted is the padagogic Seminar at Basel, (3 hours), where the students have practical exercises in teaching Mathematics, recalling the system at the Ecole Normale at Paris.

In a few of the Swiss Universities there are Seminar libraries, but the contribution made to these by the universities is so small (100-600 fr. yearly) that they are much cramped in their development. Only at Freiburg the grant is 1500 fr. a year. In one or two of the universities, there is a more or less embryonic collection of Modals, the best being at Basel.

I quote the following from the little book already mentioned, before passing on :—

"The study of Mathematics as Principal Subject is to be continued by means of the courses of lectures and the Special Lectures, as well as by work in the Seminar. For the study of Mathematics as a Secondary subject, the following are essential :—

The preliminary course of Mathematics and Mathematical Physics, together with two at least of the courses of lectures, at the students' choice.

Moreover the students of Physics, Chemistry and Astronomy are urged to follow the lectures on Differential Equations, Theory of Surfaces, Elliptic Functions, Mathematical Physics, and to the Mineralogists, Geologists, Zoologists and Botanists are specially recommended Projective Geometry, Theory of Surfaces, Differential Equations, and Mathematical Physics."

J. H. Graf, who quotes this advice in his Report to the International Commission on Mathematical Education "The Swiss Universities", adds "Everyone will grant that this advice of the 'Ratschlag' to students, not merely especially of Mathematics, but of all natural science subjects, is remarkable for its simplicity, its practical outlook and foresight. It will certainly have good results."

THE DUTCH UNIVERSITIES.—In each of the three Dutch states universities there are three mathematical professors, of which one teacher of Applied Mathematics and Theoretical Physics. Besides these, there are a certain number of Private Docents. The programme of the instruction during the first two years is as follows :—

ANALYSIS.—This consists of Differential and Integral Calculus and the following portions of Algebraic Analysis :—

Determinants, Irrational and Complex Numbers, Theory of Invariants, Linear Substitutions, Theory of Equations.

GEOMETRY.—In this course the Analytical Geometry, Projective or Synthetic Geometry, and Descriptive Geometry, both of two and three dimensions, are usually more or less interwoven with one another.

This instruction of the first two years is intended not only for the mathematicians, but also for the future astronomers and physicists, and a certain part also for chemists.

In the following years, besides the ordinary programme which includes Theory of Functions and Theoretical Mechanics including Mathematical Physics, considerable attention is devoted to Calculus of Probabilities, Differential Geometry and Calculus of Variations. Other subjects are also taught, at the discretion of the individual professor.

MODE OF INSTRUCTION.—There are scarcely any Dutch treatises or text-books dealing with Higher Mathematics. The books read are accordingly French, German and English, all of which languages are well known by the educated Dutch.

Viva Voce questions are frequently asked during the course of the lectures, and problems dealing with questions which arise in the course are regularly set. Everyone of the three universities possesses a collection of Mathematical models, and use is made of them during the course of the instruction.

Anyone is allowed to attend the lectures at a Dutch university. There are no restrictions. But no one can be a candidate for a university examination who does not possess a School Leaving Certificate, or the equivalent. If this certificate is that of a Realschule, in which Latin and Greek are not studied, the candidate has to undergo a complementary examination in these subjects.

In order to become a Doctor of the University, the Mathematical student must pass two examinations, and present a thesis. The first examination is taken usually after three years at the university. This first examination deals with Astronomy, Physics, Chemistry and Mineralogy, as well as Analysis and Geometry.

In the second examination candidates are examined in Integral Calculus, Theory of Functions, Applications of the Calculus of Probability, to the Physical Sciences, Theoretical Mechanics and Physics. Such a candidate often adds Astronomy.

This second examination is usually taken three years after the first examination, so that six years, or more, are usually required in order to obtain a Doctor's degree. In point of fact, the writing of the thesis is usually undertaken by the candidate in the two years after leaving the university, so that the whole period is commonly seven or eight years. This is owing to the fact that he usually obtains the post of a schoolmaster immediately after passing his second 'examinations, so that he can only write under difficulties.

The Experimental Physicist competes for the same degree, but he is excused certain portions of the Mathematics, as for instance the Theory of Functions.

The general opinion in Holland seems to be that the examinations involve too many distinct branches of knowledge.

The fact of having passed successfully the second examination above referred to confers the right of itself of teaching in a Gymnasium or Secondary School.

The candidate has already from time to time in the presence of the professor given a lecture on some subject which has suggested itself, and he has in this way acquired a certain amount of initiative.

It should be particularly noticed that, besides these two examinations, which lead, in the manner explained above to the Doctorate, as soon as a suitable thesis has been written, are the only examinations in the Dutch universities.

It should further be remarked that, before coming up to the university, the students have spent a considerable amount of time, even if they have been educated in the classical schools, in the acquisition of three dimensional ideas, and that they have studied Spherical Trigonometry, as well as the application of co-ordinates to Solid Geometry. Otherwise the programme depends on the particular school and on the particular master. The movement which in France, in England, in Germany, in Switzerland and elsewhere has led to the introduction of the Calculus into the pre-university instruction, is beginning to produce its effects in Holland, but, I believe, so far nothing has been done in this direction.

It should also be noticed that there are also certain Institutions, Technical and Military Academies, which, in a certain sense, occupy the same rank as the universities, and that these Institutions have their own programmes and systems of instruction in Mathematics. In certain cases success in passing the corresponding examination in these Institutions confer the right to teach in the secondary schools. It is also noteworthy that the Chemists, as well as the Physicists and Astronomers, have to pass examinations in Mathematics, not much inferior in difficulty in those undergone by the mathematical students themselves. The great success of the Dutch in these allied subjects may well be imputed to this cause.

ITALY

Pincherle, Sugli studi pee la laurea in Matematica e sulle seziine di matematica delle Scuole di Magistero.

The time of study at the Italian universities is 4 years, divided into two equal periods (bianniums); the first biannium has a double scope, in it the students of mathematics proper, students a minority of whom finally take the degree (Iaurea) and the majority of whom go into the teaching profession, are associated on equal terms with the Engineers. Thus the lectures have to be framed, and the other studies so arranged that they may at the same time serve as a ground-work for the former class of students, and afford the latter class the necessary mathematical machinery.

At the end of the first biannium the students who have successfully passed the yearly examinations are permitted to hold the title of Licentiate of the university. Up to 1906 the licentiate was the same for both classes of students, but now it is divided into two different parts, corresponding to the above two classes of students. For the second class of students the licentiate constitutes the right to enter the Scuola d'Applicazione, and for the first class it is the first step towards the laurea, For both classes of students it is necessary to have followed courses and taken special examinations. in the following subjects :

Physics, Organic and Inorganic Chemistry, Algebra, Infinitesimal Calculus, Geometry analytic, projective and descriptive with Geometrical drawing.

The engineering candidates have in addition Mineralogy and elementary architecture with designing (disegno di ornate).

Besides both classes of students have practical exercises in Algebra, Differential and Integral Calculus, and Geometry in all its branches above enumerated. These exercises are supervised by the assistants of the corresponding professors.

This arrangement is not found to work well as far as it regards to the teaching of Mathematics. The Mathematics required by the engineers, and the time at their disposal are such that they are neither willing nor able to accept a standard of mathematical teaching high enough for the students of pure science.

The title of licentiate, which was formerly a legal one, confers no rights on the holder of it, and the technically it is not actually a sine qua non for admission to the third year of study at the University, though it is presumably so for the laurea, and for admission to the Scuola d'applicazione.

About half of the students who enter the third year at the university are women. Indeed the profession of teaching offers every attraction to the women in Italy, whereas other professions are more lucrative and more honourable for men. A serious social question arises here. Moreover it is found that the presence of this large percentage of women has caused a degeneracy in the standards required at the University. The majority of them are mere embryo teachers, anxious to learn what is for an unmarried person a respectable competence. Only a very small percentage of the students who enter for the third year really take the laurea. A table printed in Pincherle's pamphlet shows an average of 18% (28 in 154) at the Italian Universities in the year 1908-9, for example. On the other hand the percentage of the students who remain at the university and do not pass out to the Scuola di Applicazione is still smaller, 7 to 10%.*

In the second biannium the students are obliged to follow five or six courses and some of the lectures given at the Scuola di Magistero. The subjects of the courses are Higher Analysis, Higher Geometry, Rational Mechanics, Theoretical Geodesy, Astronomy and Mathematical Physics. But unfortunately there are no lectures on all these subjects at each university. Rational Mechanics indeed is always to be found, there being a Chair of this subject at all the universities. But as for the other subjects, they are frequently conglomerated into a course entitled Higher Mathematics, or certain of them are left out of the curriculum altogether. Technically a "course" contains 50, or more, lectures.

In order to enter for the laurea, the student must have followed the course of Rational Mechanics, and at least four of the other courses, and he must, have passed examinations on these subjects. These examinations are oral, non-competitive, and (?) individual, and refer usually exclusively to the matter given in class by the professor during the year. Having fulfilled these obligations, and prepared a thesis, the candidate is put through an oral examination before a commission consisting of seven professors of the Faculty of Science and four private docents on the subject of the thesis, and on two or three other subjects of not too great length. The candidate has to obtain what in the opinion of these eleven judges is at least 70% of the possible marks; frequently the judges are indulgent.

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* —The latter of these numbers is given by Princherle; it is not quite clear whether I have interpreted right what he says, possibly this is the average who actually take the laurea, ("prosegue gli studi per la laurea in methematica").

(It is thus evident that the standard of the doctor's degree in Italy is not equal to that in Germany G. E. Y.).

It is not surprising therefore that the doctor's degree in Italy is not held in very great esteem, and does not of itself ensure a career. Of course in some cases the doctors are equal to the best in France or Germany, but the title alone does not give any guarantee. The better students, particularly in Bologna, take a fifth year before entering for the doctor examination, and have taken extra courses, and studied original memoirs under the guidance of the professors. Some of the Italian universities offer special courses for the better students, for instance the R. Scuola Normale Superiore at Pisa, an institution, with the Ecole Normale at Paris, from which it is copied, unique of its kind, the Institute Consorziale at Pavia and the seminars recently instituted at Rome and Naples.

Reference has already been made to the Scuola di Magistero. This institution was made in 1876 by one of the statutes relating to the Faculty of Science, and reformed in 1891, and 1903. The original object was double, to give the candidates for the laurea an opportunity of learning to teach, (training school) and to encourage and prepare them for research. As might be expected, these two aims could not go hand in hand, one of them was bound to oust the other, and research has in fact been ousted. The degree of Master (Magistero) given by the school is that usually obtained by teachers in the secondary schools. The lectures given at the school are stated to be on the method of teaching (pedagogy) and on the subject matter of the programmes of the secondary schools. These lectures are held usually once a week, and may be only an hour in duration. The pay of the teacher in these schools is a sufficient indication of the slight esteem in which they are held by the Government. If there are two teachers, by the statutes of 1903 they each have 300 lira a year.

The practical value of the school for teachers is theoretically insured by the statute that the students may teach and correct exercises in the secondary schools of the neighbourhood, under the guidance of the professor; practically however, this statute has remained a dead letter.

The Scuola Normale at Pisa was founded in 1846 on the model of that at Paris. The alumni, who are University students in science and letters are admitted by a special examination, and receive either free board and lodging (posio gratuito in convitto) or a scholarship in money. They are obliged to follow courses in the school itself, as well as their University courses. The internal courses deal with foreign languages and matters complementary to, and often not represented in the University programme; for example the theory or numbers, substitutions, complementary notions on differential equations, etc. The frequency of their attendance at lectures is not sufficient for these students, if they have not also a good report (votazione) they lose their places as "normalist." This school is unique in Italy, and gives a special character to the Faculty of Science at Pisa.

The Consorzio at Pavia, also referred to above, has established an internal course of the candidates for the laurea in Mathematics, and provides a Professor in charge of this course and special lectures. In certain Universities there are prizes for the best doctor dissertations, (Fondazions Corsi at Rome. The Vittorio Emanuele and Merlani prizes at Bologna). Pincherle makes some interesting remarks on the reasons why the number of students in the Third-year of the University is so much greater than the number who enter for the laurea. Some of these students have failed in the licentiate, for, as already remarked though it is one of the statutes that this should not be allowed, in practice the rule is broken at many of the Universities, and this with the consent, more or less veiled, of the

Government. Thus a certain number of the students of the Third-year are those who would have entered the Scuola but are unable to do so. Instead of repeating the Second-year, as they ought to do, they prefer to go on to the Third-year, so as to keep their title of student, and perhaps to make their parents believe they have not wasted their Second-year. Thus for instance in the year 1902-3 and in 1903-4 the number of students at the University of Genoa in the Third-year was 81, and the number entering for the laurea was in no year more than 4, Princherle says those Third-years were the refugium peccatorunm of those who has failed to enter the First-year of the Scuola.

A Certain number of those who have entered the Third-year give up Mathematics before proceeding to the laurea, finding it too severe a discipline for them. The result is the same whether the reduction of numbers is due to this reason or to the former ; the supply of adequately prepared teachers of mathematics in the schools has not increased, whereas the demand has multiplied many times. Pincherle states explicitly that the great increase in the number of aspirants to the engineering profession and the comparative decrease in these for the teaching career is due to the difference in the economic and social conditions offered by the two careers.

On the other hand, as Pincherlé says, the career of a teacher appeals to the less ambitious desires and ideals of the female students, and on the other hand, they do not have the same competition in this as in some other professions. A female student who has attained the laurea has opened to her the posts at the so-called Normal schools, which are secondary schools ranking with the liceums and technical institutes in rank and in salary, and open only to women. This accounts for the large percentage of female students. The effect is to the detriment of the scientific character of the University, and the intensification of the purely professional aspect. He adds, "When large proportion of the auditors are led to attend the lectures only with a view to obtaining a diploma, as an opening to a career which is considered desirable, the scientific level of the teaching is bound to be reduced."

It is undoubtedly one of the problems of the mathematical school at any University to prepare adequately the teachers at the secondary schools. In former days it was considered sufficient to give him a good education, and as soon as he got his post, he had to find out for himself the best method of teaching, the best curriculum and the best textbooks. It is not surprising that the results were somewhat heterogeneous. In the last twenty years the foundations of geometry and analysis, and the various possible ways of teaching elementary mathematics have been lifted and resuifted. Theoretically at least the budding school master ought to have an opportunity of becoming acquainted with the main lines of these researches. We have seen that the Austrian mathematicians considered this point seriously, and pointed out the great difficulties in the way of teaching properly the foundations of

Geometry. In Italy probably some of the difficulties found in Austria may exist in a lesser degree. Probably the Italian mind is riper and more acute at the age to be considered than the Austrian. Be that as it may Pincherlé advocates increased opportunities for studying elementary Geometry at the University from the scientific, critical and pedagogic point of view.

Another point on which Pincherlé insists is the inadequacy of the tests applied for the distinguishing of the candidates for the laurea. These are all oral examinations with the single exception of the thesis. One may easily ask whether the time spent on the thesis is not often out of all proportion to that spent by the candidate on acquiring ordinary information. Few indeed are the theses in any language which remain on the roll of honour in mathematical science. Generally the candidate owes much to the advice and conversation of others in the elaboration of his thesis. Pincherlé maintains that it is too much to demand that the thesis should have scientific value, but it is not too much to require the candidate to know thoroughly the principal points in the several branches of his subject, and should be able to show this mastery in face of simple questions in algebra, analytic geometry, infinitesimal calculus and mechanics.

Pincherlé accordingly formulates the following demands for reform :

(1) That from the beginning the teaching of mathematics should be different for the aspirants to the laurea and the engineers;

(2) That a written examination should be added to the oral one in the case of algebra, analytic geometry, projective and descriptive geometry, infinitesimal calculus and rational mechanics;

(3) That no one should be admitted to the Third-year of the study at the University without having passed all the examinations, oral and written, of the first biannium ;

(4) That the second biannium should comprehend the following :

(a) Fundamental courses, obligatory and fixed, on rational mechanics, theory of functions, complements of geometry and physical mathematics;

(b) Complementary courses, intended to give a higher mathematical culture and turning on special subjects in Analysis, geometry, mechanics, astronomy, etc.

(c) A scientific seminar in which, under the direction of the professors, the more important classical works and recent memoirs should be read and commented, and the students should be led by appropriate exercises to the production of scientific monographs;

(5) That all through the four years, attention should be paid to the preparation of the future teachers in the school; on the one hand by special courses of elementary mathematics under the threefold aspects :—

(a) The revision of the matter already studied at the elementary school;

(b) Pedagogy and the theory of method;

(c) The connexion of mathematics with the other branches of science.

And on the other hand by lectures, discussions and reports, given by the students themselves on the lines theoretically prescribed at present by the Scuola de-magistero.

In the last year it might be possible to introduce that apprenticeship in one of the secondary schools of the neighbourhood indicated in § 16 of the Regolamento Orlando for the Scuola di-magistero, supposing the difficulties of realisation not too serious.

(6) That, during the four years, the students should be advised to attend other courses, tending to enlarge their mental horizon (science, biology, philosophy) and that they should give proofs of the culture so acquired by giving lectures or reports, to which reference is made in N^{o} 7;

Again, at the conclusion of the course of study, the candidates should be expected to show a sufficient knowledge of foreign languages;

(7) The permission to persue the studies of the third and fourth years should be given in two distinct manners :

(a) By means of a laurea scientifica. This is to be obtained, after the trial lecture above referred to (§ 16), by the presentation of a thesis, original in results or in method and printed and published, and afterwards by a discussion of the thesis, and of some minor subject in an oral examination. The aspirants to become private docents of the University ought to have passed the laurea

scientifica. The candidates for the laurea scientifica should have attended the courses referred to above (N°.4, a and b 6), and the scientific seminar during the second biannium.

(*b*) By the laurea didactica. After a trial lecture of a scientific-didactica nature, this is obtained by producing two written papers, one on methodology, and the other on a question of analysis, geometry, mechanics or mathematical physics, to be finished in a certain determinate period of time and according to rules to be specified; besides this, there should be a discussion on certain chosen themes in an oral examination. The aspirants to the laurea didactica must have attended the fundamental courses of the second biannium mentioned in (N°.4, a and 5). The examination for the magistero, as at present constituted, shall be abolished. The laurea didactica shall be obligatory for eligibility for teaching posts in the secondary schools.

RUSSIA

There are eleven universities in Russia, the oldest and most important for mathematicians is that of Moscow, founded in 1755. That of Petrogard was founded in 1819 and that of Warsaw in 1869, the latter has a real mathematical school at the present day, owing to the presence of ProfessorSierpinski; it is, of course, properly a Polish university, and the memoirs of Sierpinski and his pupils are published in Polish, with, usually, a short summary in French.

The newest universities of Tomsk (1888) and Saratow (1909) have no Faculty of Mathematics and Physics. The remaining five universities are the following :—

Juriew (formerly Dorpat) (1802).
Kazan, (1804).
Kharkow, (1805).
St. Wladimir in Kiew, (1834).
The University of New Russia in Odessa, (1865).

Considering the size of Russia, its advance of late years, and the ardent love of study shown by its students, this list of universities is, of course, pitiable, and one cannot be surprised at the enormous number of Russians who are found at universities outside their native country.

It is interesting to notice nevertheless the high plane on which it is attempted to keep the studies of mathematics at the universities. There is nothing of that spirit of utilitarianism which underlies the English system of examinations. Here as elsewhere on the Continent of Europe, a high mathematical ideal is maintained. In a little official pamphlet entitled "Renseingements Sur Ie plan d'études de la Faculté physico-mathématigue de I'Université de St-Petersbourg", (19) (8) we find the following:

"Each Faculty of the University tends to realise, as far as may be, the following ideal in the teaching of Science :

"Not only to communicate in a proper order a certain quantity of information respecting the facts of Science to the students, but also to develop the methods and the intellectual processes which have led to the discovery of the fundamental truths of Science, which may serve for the research of other truths still unknown; to teach the students the means of gaining possession of these methods and to be able to apply them in solution of the various questions, suggested by their own studies; to exhibit up to a certain point those essential things which have already been effected in Science, and to throw light, as much as possible on the path it is pursuing; to indicate new questions and problems, as they present themselves in the gradual development of Science; in a word to develop in the students on the basis of a certain amount of acquired knowledge, the spirit of scientific curiosity, and invention and the capacity for scientific work."

In the light of this it is interesting to note that there is at present moment a strong young mathematical school in Russia; more than one piece of original mathematics which will live, has been produced by this school, and it is to be hoped that its influence will spread among that highly intellectual people.

In Russia there is a Faculty called the Faculty of Physico-mathematics; it is divided into two sections, that of Mathematics, and that of Natural Science, and corresponds therefore to the Science Faculties at the other European universities. All the students of the Faculty have to take the course of General Physics and that of General Chemistry, lasting one academic year; otherwise the studies of the two sections are different.

As regards the Physical Section, the Russian scientists have for some time recognised the importance of a mathematical groundwork for their students, and in particular the essential character of the calculus and of Analytic Geometry for them. At the universities of Moscow, Petrograd, Kiew, Rharkow and Odessa accordingly, short courses of Mathematics for Science students have been introduced, and in Moscow and Petrograd there are also courses of Elementary Mechanics for the students of the Physical Section.

The length of these courses is different at the different universities. At Petrograd it is 3 hours a week for two years for Mathematics, and 2 hours a week for one year for Mechanics, the complete time of study at the university being four years.

For entrance into a Russian university, Latin is compulsory, and must be taken in a complementary examination by students of Secondary Schools other than the gymnasia (8 years of study), or the Imperial Alexander Lyceum, or the Imperial School of Law. A very few other students of mathematics have come up from the ecclesiastical schools called Seminaries very badly prepared, and have had to go through a complementary examination in Mathematics, Physics and Modern Language. The average age of the freshmen is 18-20.

This compulsory Latin keeps away most students of mathematics who have not taken Classics at school; the others go for the most part to the technical schools. In the Gymnasia there are special classes for the top boys in Analytic Geometry and Infinitesimal Calculus, as in France, but the level of knowledge of these subjects is not high, and, as in the French Universities, classes have been formed at the universities for first year students in these subjects, including also higher Algebra. It is nevertheless found in practice that there are still gaps in the students knowledge which have to be filled up, and the Russian mathematical professors have to spend much time and thought on this matter. In spite of all this care, many of the students find that they are unfitted to follow mathematics seriously, and they change Faculties or leave the University altogether. There is a crying need of improvement in the secondary in Russia, and the Minister of Public Instruction is in course of introducing drastic changes so as to bring the secondary schools into line with those in Germany and France.

The programme of the Mathematical Section of the Physico-mathematical Faculty was entirely revised in 1906 before that the two sections were obliged to follow one and the same course of study. Now the programme of the section is divided into three portions:

(1) Mathematics and Analytical Mechanics;
(2) Astronomy;
(3) Physics sub-divided into two parts, *(a)* Physics and *(b)* Physical Geography and Metereology.

The student may choose one or more of these parts; moreover he may, if he please, also take portions of the programme of other Faculties. In his principal subject he is obliged to follow all the courses in the portion in question.

Appended is the programme of the University of Petrograd, the divisions have reference to the examinations to be discussed further on, and the numbers imply the number of hours in the two respective semesters devoted per week to the subject in question.

The examinations of any one division cannot be passed until those in the division below it have been passed. The voluntary courses will not be required in the examinations. The courses of Higher Geometry and Descriptive Geometry take place in turn every two years. Beside the courses above mentioned, there are others, which are not given every year, for instance :

(1) Abelian Integrals; (2) Theory of surfaces; (3) Analytic Theory of Differential equations; (4) Vector analysis; (5) Historical and critical review of the principles of Mechanics, most of which courses were given by Prival docents.

The programme of the University of Moscow, where the well-known mathematician Egoroff has developed a rising school, is not dissimilar and is accordingly not reproduced here; it is to be noted, however, that at Moscow there is a seminary at which interesting subjects such of term-by-term integration of series is discussed, and the newest results brought before the class.

Programme of the Mathematical Section at Petrograde University

I. Mathematics. II. Astronomy. III. Physics.

First Division

Obligatory courses for I, II and III.

Analytical Geometry, 4, 4; Spherical Trigonometry, 1, 0; Introduction to Analysis, 4, 0; Differential Calculus, 0, 4; Descriptive Astronomy, 3, 3; Physics of molecular forces and Heat, 3, 3.

I

Higher Geometry, 1, 2.
Descriptive Geometry, 1, 2.

III

General Chemistry, 4, 4.
Crystallography, 2, 0.
Physical laboratory.

For I, and II the course of General Chemistry is recommended.

Second Division

Obligatory courses for I, II and III.

Higher Algebra, 3, 3; Applications of the Differential Calculus to Geometry, 4, 0;

Integration, 350; Applications of the Integral Calculus to Geometry, 0, 3; Spherical Astronomy, 3, 0; Optics and Acoustics, 2, 2; Electricity, 2, 2.

II

Practical work in Spherical Astronomy, 2, 0.
Practical Astronomy, 2, 0.
Practical work at the Observatory, 0, 2.

For III the course on Practical Astronomy is recommended.

Third Division

Obligatory courses for I, II and III.

Definite integrals, 2, 2; Ordinary Differential Equations and Calculus of Variations, 3, 3.

Statics, 0, 2; Kinematics, 2, 0; Particle Dynamics, 0, 3; Rigid Dynamics, 4, 0; Hydrostatics and Hydrodynamics, 0, 3.

I

Partial Diff. Eqns., 3, 3;
Finite Differences, 2, 0;
Probability, 2, 2;
Elliptic Functions, 3, 0;
Theory of Numbers, 0, 4.

II

Partial Diff. Eqns., 3, 3.
Theoretic Astronomy, 3, 0;
Geodesy, 0, 3 ;
Celestial Mechanics, 0, 3;
Higher Optics, 2, 2.
Thermodynamics, 2, 2;
Partial Diff. Eqns., 3, 3;
Electricity, 4, 4;
Metereology and Physical Geog., 3, 3 ;
Terrestrial Magnetism.

The following are recommended :—

I. Solution of problems in Mechanics, 2, 2; Theoretic Astronomy, 3, 0; Celestial Mechanics, 0, 3.

II. Probability, 2, 2 ; Ell. Fns., 3, 0; Problems in Mechanics, 2, 2; Figure of the heavenly bodies, 2, 0; Tides, 0, 2.

III. Probability, 2, 2 ; Problems in Mechanics, 2, 2 ; Elliptic Functions, 3, 0 ; Theoretic Astronomy, Metereology, 3, 3.

The system of examinations is not, and never has been, satisfactory in Russia. At present there are grave differences between one university and another, but, on the whole, one may say that the new French system of certificates has been introduced, whereby the students are able to take examinations in each subject separately, instead of the old system of two examinations, one at the end of two years, and one at the end of four. The results are so unsatisfactory that it is hardly necessary to enter into the subject except as a warning. A great difference was made between examinations on the obligatory courses and those on the optional subjects, but the latter had degenerated into mere formalities. Moreover the examiners being the professors who gave the courses, the examinations were no check and no guide to the teaching; the same cause rendered the differences from one university to another overpowering. An attempt to graft the German idea of the Staatsexamen to the Russian system was not more successful.

The new system was introduced in 1906, but in many of the universities they seem to have returned more or less completely to the old system; the examination at the end of the second year has reappeared. At Mosoow, Petrograde, Khyrkow, Kiew and Odessa the French system, introduced in 1906 works at the present day, but not very satisfactorily. The French organisation has only partly been adopted, and the German idea of the Staatsexamen has been retained. The system of divisions, already indicated, is such that the examinations in one division cannot be taken unless those in the lower divisions have been passed. The student may choose what subjects he pleases and make the examinations practically when he pleases; these are held so often that they may be held every fortnight, at the will of the professor who fixes days for the examinations. Theoretically if the student fails, he cannot go in again in the same semester, and other rules of the same kind have been made, practically such rules are not regarded.

If the student has kept his 8 semesters, and then entered in the German way in his book, he obtains his leaving certificate, admitts him to the State examination, provided he has passed the examinations of the first and second divisions. The examinations of the third division count as the State examination, and give him the right to teach as in Germany.

In Moscow examinations only take place three times a year. One reason, besides those indicated why the new system does not work very satisfactorily, is that the Russian students are far less well-prepared and far less independent than the French, so that the liberty of choice accorded to them if not always well used.

The Russian universities accord the degree of Master of Science, on the presentation of a satisfactory thesis, and the passing of an examination. These examinations play an important part in university life, since no one can hold a professorship at the University, or a Theoretical Chair at one of the Technical Schools without having this degree. For a university post, this is not yet sufficient, the candidate has also to have the degree of Doctor of Science, requiring the presentation of a second thesis of higher order than the first, and its public discussion. Both these thesis have to show signs of original work.

At Petrograde the examination for either of these degrees bears on the following subjects :—

(1) Higher Algebra; (2) Differential and Integral Calculus, with applications to Geometry, (3) Integral Calculus, Partial Equations, Calculus of Variations; (4) Theory of numbers; (5) Elliptic Functions; (6) Finite Differences and Probability; (7) Analytical Mechanics.

The examination in these subjects need not take place on one day, but must take place in one year. The professor appoints an evening on which there is a Faculty meeting for each examination, and the examination takes place in the presence of at least one professor besides the examiners. The order in which the examinations is taken, as well as the intervals between them are left to the choice of the candidates.

When the candidate is going in for an examination, he calls on the professor, as in Germany, and finds out from him the broad lines on which he is going to be examined. At Moscow the candidate presents a list of the works he has studied. If the Faculty is satisfied with this, it determines the range of the questions set; if not, the list is returned to the candidate, with indications of the required improvement.

If the candidate fails, he is not allowed to go in again for a year.

CALCUTTA UNIVERSITY

REGULATIONS FOR THE M.Sc.

The first regulations sets a limit of time. "Any candidate who has passed the B.Sc. Examination not less than 2 academical years previously, provided he has prosecuted a regular course of study in a College affiliated to the University," or "not less than three academical years" otherwise.

This puts the degree of M.Sc. on the plane of the old-fashioned M.A. of Oxford and Cambridge, which marks a certain age and status, and the facts that the man has been at the University, and taken his B.A. There is nothing corresponding to this in France, but in Germany the Doctor's Degree cannot be taken without four years at the University, and the same is same for the Staatsexamen. In Italy also the period is four years for the Doctor's Degree, and two years for the licentiate.

REGULATION 6. ON THE PRACTICAL EXAMINATION

There is a practical examination in all subjects except Mathematics. In France there is a practical examination for the Licentiate, it lasts two hours for each certificate.

The Calcutta Examination is entirely a written examination. This is the case with no examination on the continent. It consists only of answering printed questions, without books and in a definite time (4 hours, eight papers).

If we compare this with the Licentiate in France, where also we have for each certificate a four hours time paper behind closed doors, we see that the results of each such examination are checked by two subsequent tests, one practical lasting two hours, and the last oral. Each of these has to be passed before the next is entered on. The Licentiate requires three certificates, but they need not be all gained at the same time. Here there are eight subjects, and they all have to be taken at the same time. This makes a serious and quite unnecessary demand on the memory, and also tends to make the students trust to one is overworked. The Licentiate is generally taken after two years only, though there is no limit as to time, but the principle is not affected by this fact. If we compare this examination with the demands for the Doctor's degree or for the aggregation, the same remarks will apply.

If we compare with Germany, we naturally take the Staatsexamen. Here we have an examination which is confessedly unsatisfactory, and for which the candidate commonly overworks. The programme is too burdened. The time-papers are however here only two, though take on one day, and are subsequent to the presentation of the thesis or theses, and the examination does not terminate without a viva voce examination in each subject taken.

If however we examine carefully the subjects as set in the Calcutta Calendar, and compare them with the Programme of the Licentiate at Paris, we find that the difference in the number of subjects is not so great as would appear. Thus at Calcutta Differential and Integral Calculus, Differential Equations, Calculus of Variations, Geometry of Surfaces (Theory of Functions) are each counted as a separate subject, while at Paris they are all included under the one, "Differential and Integral Calculus." The remaining subjects at Calcutta :

Higher Algebra.
Higher Plane Trigonometry.
Spherical Trigonometry.
Theory of Equations and Algebra of Quantics.
Plane Analytical Geometry, and Higher Plane Curves.
Theory of Groups.
Finite Differences and Calculus of Functions.
Vector Analysis and Quaternions.
Projective Geometry.
Theory of Numbers.

One subject only of the last five being taken by each candidate, unless they take Theory of Functions, which is an alternative subject at Calcutta, but obligatory for the Licentiate.

At the top stands the note "It is to be understood that in the following list of text-books wherever a book is recommended for a subject, students are expected to read those parts of the book only which deal with that particular subject."

This saying is totally at variance with all proper ideals of university work. It reminds one of Luther's old quarrel for the freedom to read the Bible. A proper student ought to be free to read all and everything he can lay his hands on, exercising his individual judgment as to the parts to be passed over rapidly or omitted. To dictate chapter and verse is to place the university on the level of a school. Many a time a brilliant discovery has been made by reading a neglected part of a book or an obscure memoir. To inculcate from head quarters the practice of reading only set portions of text books is most dangerous.

If we consult the report of M. Tannary on the Ecolé Normale at Paris, we find quite a different spirit. The young men come up from school, having just gone through a severe competitive examination, for which they have been in training. Here is what he says :

Extract from a letter of Professor Pincherle's, dated November 29, 1914.

(1) Il n'y a rien de changé dans nos régLements depuis, 1911.

(2) Nousavons institué un Séminaire a Bologne dans le genre de celui de Rome; c'est a dire que des professeurs de la ville et meme d'autres Universités y tiennent une conférence par semaine sur des sujets variés de mathématiques.

Le résultat est satisfaisant.

(3) La surveillance du Gouvernement sur les professeurs de l'université exsiste potentiellement ;

mais en effet elle ne s'exerce quere. Cependant Ii minimum des lecons par année est de 50.

(4-5) Le Ministere ne peut prendre de décisions de quelques importance, soit pour le reglement des Cours, soit pour le personnel, sans en saisir d'abord le "Consilio Superiore dell, Istruzione pubblica", formé en partie de professeurs, en partie de parlementaires.

(6) Le Laurea est nécessaire pour les places de professeur, meme secondaire, et sans elle on ne peut entreprendre la carriere de l'enseignement.

(7) Pour entrer au service de l'Etat en outre Ia Oaurea, on doit faire un examen de "Concorso", selon la qualité de l'école ou l'on aspire.

(8) Il n'y a pas eu de changement dans l'election des professeurs dans ces derniers temps.

En premiére année donc, les éléves de l'écdle normale supérieure suivent, a la Sorbonne, le cours de Calcul différentiel et intégral (deux lecons par semaine, pendant les deux semestres). Ils ont deux conférences de Mathématiques á l'Ecole par semaine ; ces conférenconsacrées surtout á des interrogatoires sur le conr et a des exercices. Outre que le cours de la Sorbonne est parfaitement adapté a leurs besoins, il n'y a pas lieu de chercher a pousser plus loin les élèves qui doivent, pendant cetue année, consacrer beaucoup de temps à la physique D'ailleurs, quoique, dans cette premiere année, les élèves aient d'autant plus besoin d'étre guidés qu'ils ont été soumis, au lycée, a un entrainement plus factciie, l importe de les habituer un peu a cette liberté qui est necessaire au developepment de l'initiative, de leur laisser du temps pour lire ce qu'ils veulent, pour refléchir a un point qui les interesse, plus longtemps qu'il n'est indispensable, enfin pour chercher a resoudre des problemes qui ne leur sont pas imposes.

AN INAUGURAL LECTURE DELIVERED BEFORE THE UNIVERSITY OF CALCUTTA, DECEMBER, 1915.

By W. H. YOUNG, Sc.D., F.R.S.,
Hardinge Professor of Mathematics, etc., etc.

It seems fitting, as I have recently returned from a somewhat extended tour through America, China and Japan, that I should begin my lecture to-day by a few reflections which, if not directly suggested by that tour, are in close connection with it. Briefly I may say that I have returned to India even more convinced than I was when I went away, that the future is to the thinker and that action is bound to become more and more a simple corollary to thought.

Even up to the end of the nineteenth century the position was tenable that it was better even for the leaders to err on the side of practical knowledge than on that of theoretical equipment. But everything I have seen and heard has borne in on me that the contrary is now the case, that now though both are necessary, it is better to have rather too little practical knowledge than too little power of dealing with the concepts which alone render practical knowledge vivifying and fruitful.

It is neither the time nor the place to elaborate the grounds which I have for taking up this position. It is sufficient to agree one fact known to all the world. The time element has in this sense come to be a permanent constituent in our conception of the universe, that we cannot think of it except as a changing one, as an assemblage of events and not of stationary parts. The speed with which one can move from country to country has its analogue in the rapidity with which each separate portion—if any portion can be said now to be separate—of the earth's surface is itself undergoing modifications in the mental and moral characteristics. of its inhabitants and of the system under which they are living. Indeed, I am credibly informed that serious efforts are being made to solve the problem as to how the physical and intellectual characteristics of a certain prominent Asiatic people may be suitably influenced—even to the extent of trying to modify the shape of their skulls.

In this changing universe law and order still have their place. Codified knowledge is still required and still desirable, and most of us must stop there. But the highest place is reserved for him who can consciously or unconsciously adapt himself to changing conditions, who can remain in touch with growing knowledge, who can shape that knowledge to the ends of an ever higher civilization, can himself rise to creative acts. The coolie can, if he is properly taught, learn to embody in concrete form after the approved fashion what the highest intellectual resources of the race may have been needed to conceive. But there is no longer any finality : what the aeroplane is to-day it will not be tomorrow, and the people which rests on the achievements of to-day will be overtaken and passed before the night is over.

The heritage of the future is for the thinking races.

How does this bear on India ? And how is it connected with the subject on which I have elected to speak to you this session ? The best people in India have always been thinkers; the Indian then starts with a handicap in his favour in the race which the nations of the world have to run. We are then justified here in looking forward to the future with great hopefulness. On the other hand his thinking has for the most part related to a stationary universe : in India then even far more than in Europe it is essential that the changing character of the modern world should be brought home to its best minds, otherwise the Indian will have to set against the balance to his credit, to which I have just referred, serious disabilities. He is like a man who has money in the bank and does not know how to use it, because he does not understand the conditions which are governing the commercial world round him.

We see then the need for earnest and intense effort, if we are to bring home to the thoughtful Indian the necessity for doing everything in his power to encourage what is understood by the word *research* in its highest form. By this we do not mean that laboratories must be created and equipped; at least we do not mean this exclusively nor even primarily. Laboratories suggest to some chemical experiments; to others nothing more than bottle-washing. But even those who understand and appreciate to the highest degree the function of laboratories in the modern world, even these often fail to realize the fact that laboratories only represent a part of what is characteristically modern in intellectual activity. It would be a misfortune for any people if with it research were understood to mean laboratories and nothing more. In every department of human effort and mental activity attention must be concentrated on the time elements, on the formulation and solution of new problems, and on the attacking of old problems by new methods. Otherwise only a small section of the leaders of the nation will be qualified to take part in the new developments which every rising sun may now be said to bring with it. I would go further and would say that the highest intellectual work of the kind I have in view cannot be done in the laboratories, though some of it may be done outside by men who actually or virtually direct the labours of those whose chief interest lies in the actual carrying out of experimental researches.

I am, I may say, convinced in my own mind that India needs, for example, a mathematical institute properly equipped and staffed, far more than it needs a new laboratory. It is that in such a laboratory successful experiments may result in the temporary material amelioration of a section of the population. What is needed far more even than this, is that Indian leaders of thought should be produced in sufficient quantity and of the right quality. You will then ask for and obtain what India needs most. The right experiments can only be made when the right kind of thinking has determined the subjects with which the experiments are to deal. For this not mere technical knowledge but broad and independent thinking, leading as it must do to proper preliminary enquiries, is the most essential requisite. Where so much needs doing, where so many new possibilities present themselves, the need above all things is for the thinker who has all his thinking life been in contact with change and growth, to whom progress, the solution of difficulties as they arise, and creative acts of the intelligence become almost matters of routine. The man who has spent his time in developing the experiments of others or in painfully acquiring a knowledge of the results at which others perhaps long since have arrived, is not a thinker. Even in England our specialists have too often been men of this type, with the result that it is a common proverb in Government circles that specialists are always prejudiced, that they cannot take a wide view, that they were born to be servants. The explanation is a simple one : England's success in the nineteenth century was so great that she finds it difficult to abandon nineteenth century methods; and in her Universities real research in the best sense of the word has a secondary place, and in some is all but stifled by the examination system. A country is apt to have the experts it deserves. I do not need to tell you of the new spirit which is abroad all over the British

Empire, and which is now transforming old ideas into new, and which makes us confident that this reproach against our experts will soon have no justification in fact.

I cannot in these few words hope to have convinced the sceptic; I should be satisfied indeed if I have made my point clear even to a portion of my most sympathetic hearers. Rest assured, however, that I am myself absolutely convinced that the man educated on old-fashioned lines, equally whether this education was practical in tendency or dealt largely with traditional learning, will not be fitted for living in the new world which is becoming. To be able to think even is not enough; it must be the man who thinks without effort, to whom thinking is a natural process, who is, so to speak, going on thinking all the time (the man who at most thinks about his job and finds thinking at other times an intolerable effort, is not adequate) who alone will be capable of dealing with new conditions as they arise, who will have the imagination to foresee undreamt of possibilities of which the past offers no examples. The man of the future must, I say, be above all things a thinker, and a thinker of the dynamic type whose subject of thought is a universe of events and not one of things.

My reasons for talking to you about the Theory of Functions of a Real Variable are :—

(i) It is a new subject introduced into your curriculum.

(ii) It is a living subject to-day, growing and likely to grow for some time to come; so that

it does not yet belong to the codified type of mathematics. Accordingly anyone studying it, has impressed upon him the fact that there is no such thing as absolute knowledge, that there are always problems which are unsolved at any given moment, but which will certainly be solved. A student in such a branch will certainly in the course of his training, come across problems which, when he first hears of them, are unsolved, which as time goes on are being seriously attacked, and which, before he has completed his studies, are solved. I he is really an able man, he may himself take a part in such it solution. In any case, his attitude of mind toward the innumerable problems which present themselves in ordinary human affairs, will not be that of a fatalist; he will have no use for the man who explains to him more or less elaborately that such problems are necessarily insoluble.

(iii) It is not only a living and a growing subject, it is a subject which is particularly characteristic of the twentieth century. The advances made in it are without a doubt the most remarkable advances which mathematical science has to show in the twentieth century so far, and it accordingly finds its adherents all over Europe.

(iv) Its results, and more especially its leading ideas, are of fundamental importance in that branch of mathematics which in the last half of the nineteenth century held the field and still has many disciples, as it is bound always to have, *viz.,* the Theory of Functions of a Complex Variable.

(v) It is almost certain that it is the most admirable example in mathematical science, which is itself a science of pure thought, of the creative power of the human mind. The steps taken in its elaboration at once dwarf and render intelligible earlier steps in the progress of the science. No more sharpening process can be imagined for the intellect than that which its study requires. It deals, too, with the very latest of the new sciences which the human mind has created for itself. For the first time that which is in itself a mere negation, the finite or the non-finite species of the impossible, has been yoked to the car of intellectual progress and rendered completely subservient to the needs of the intellect.

(vi) I may perhaps be allowed to add as a further reason—not so much the fact in itself that I have actively occupied myself with researches in the Theory of Functions of a Real Variable, but rather the connected circumstance, that the somewhat numerous papers which I have written on the subject are almost all of them in the English language and are published in periodicals which should be easily accessible to you. These papers are, for anyone who has the time, energy and will power necessary for their systematic study in chronological order, a convenient means of approach to the very heart of the subject.

I give these as my reasons for talking to you about the subject, and I hope, when I am gone, that they will be regarded as reasons why it should continue to be studied, if possible, in an ever-increasing degree. Some present may feel daunted with the prospect of the hard thinking which may be involved; to such I do not address myself. Nations which have become great, have owed it to their possession of only a few great men and to the willingness of others to follow them. It is those who would fain, would become *leaders* that I have in mind. Such men must be capable of sustained mental effort, and the training in accurate thinking and in the attacking of new problems that I am now advising for them, is the best purely intellectual training which in the present state of human intellectual progress can be conceived. (Even the older mathematics has been acclaimed by the best lawyers both in England and in India as the study which best prepares a student for the study of law. The legal system, however, is even in the British Empire, more or less codified, and we may use a proportion and say that, as codified mathematics is to the study of other codified knowledge, so is growing mathematical science in its subtler forms, to the investigation of the various problems of human life and learning in all their complexity and difficulty. The growing mathematical science exercises in fact those additional faculties of the mind which codified mathematical science does not.) But it would lead me too far afield if I were to attempt now even to enumerate reasons for the study of mathematics as I here understand the term. I can only say that the more distinguished men of science and learning whom I have met, are expecting a magnificent future for mathematical studies, and it is but slight exaggeration to say that some of them are even now all but counting the days to the time when they will be able to work hand in hand with the mathematician, and this not by his descending to their non-mathematical level, but by their rising to his.

Calcutta. W.H. YOUNG.